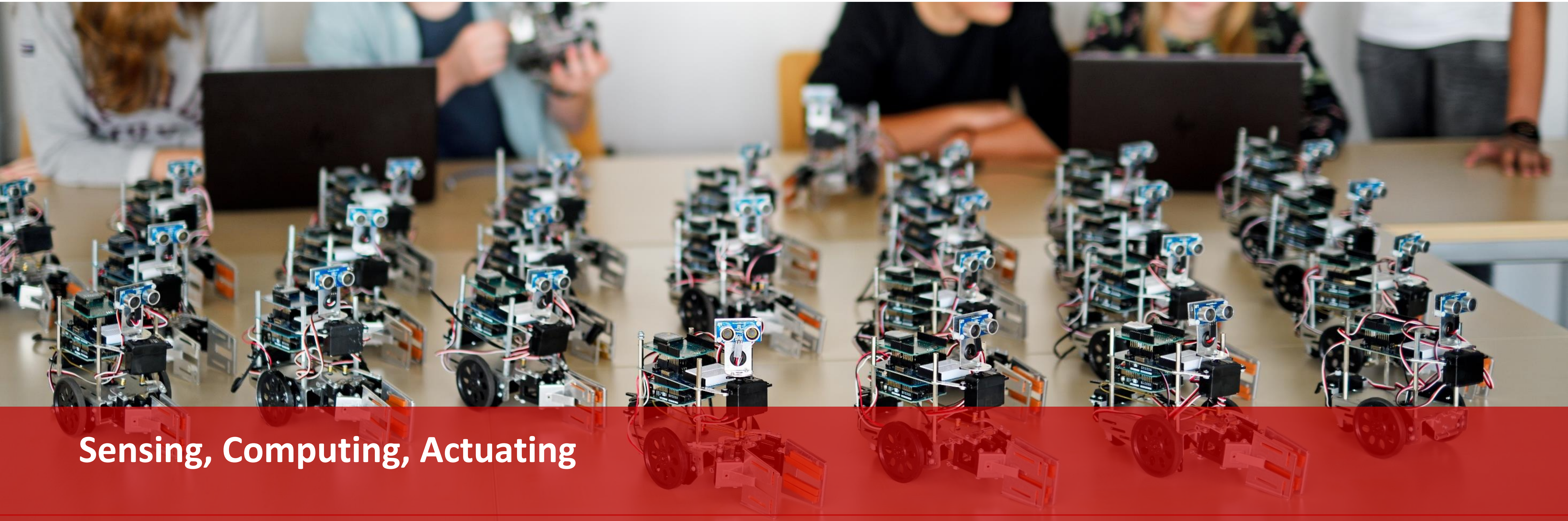


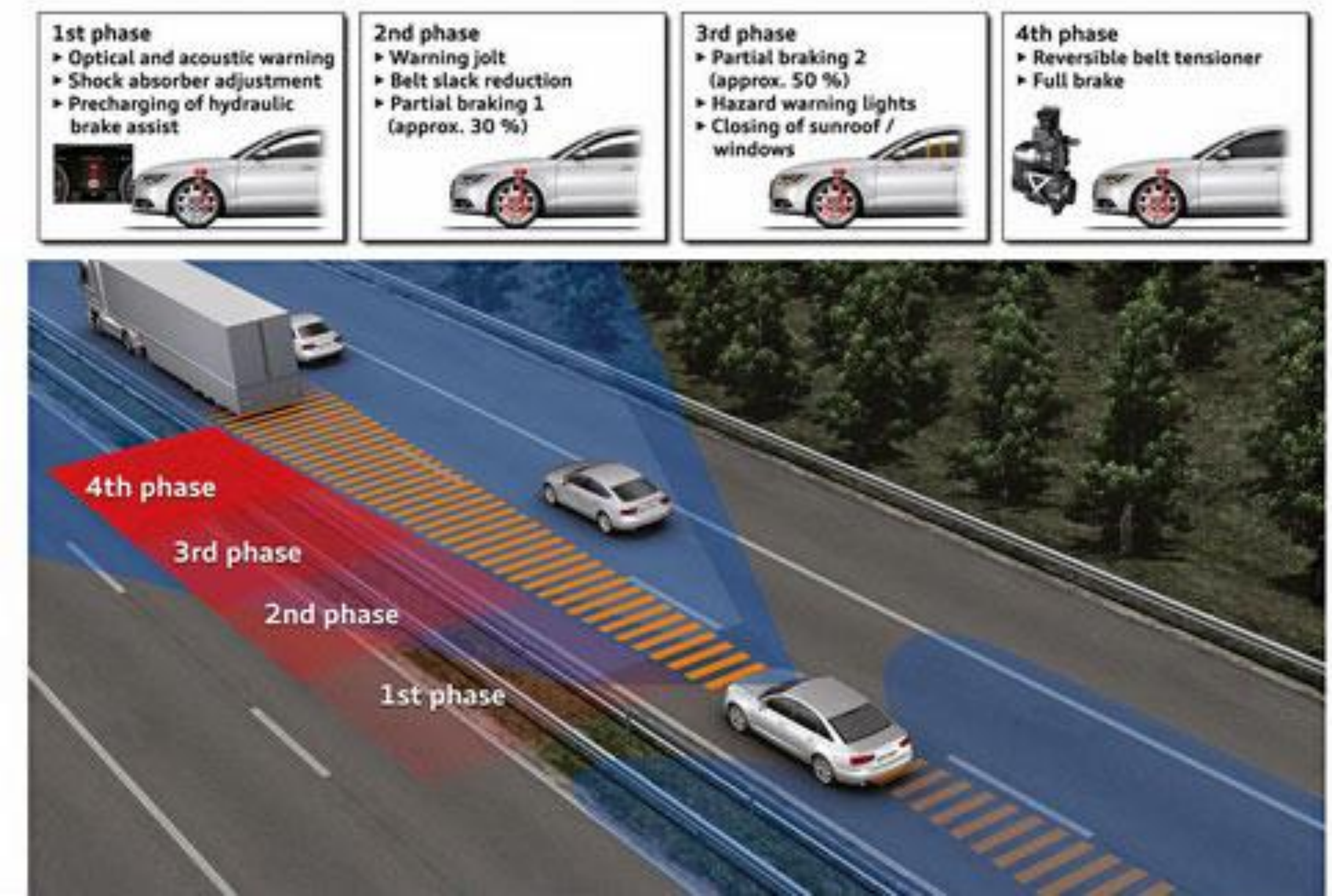
Sensing, Computing, Actuating

Sander Stuijk (s.stuijk@tue.nl)

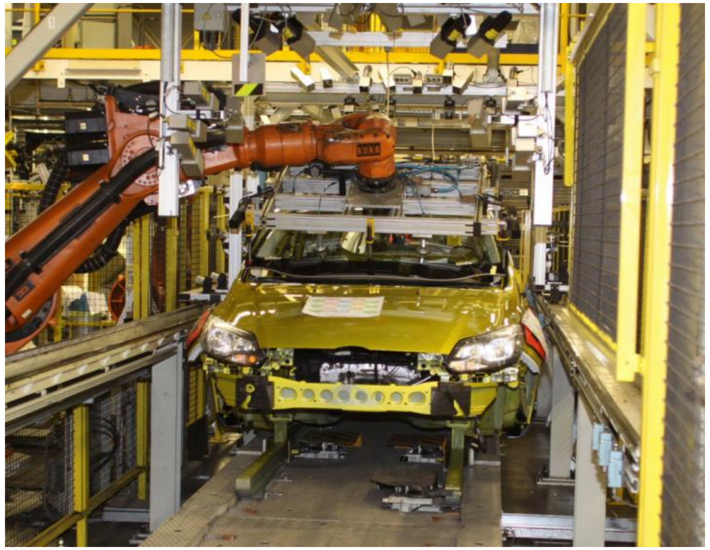
CONTROL AND COMPUTATION

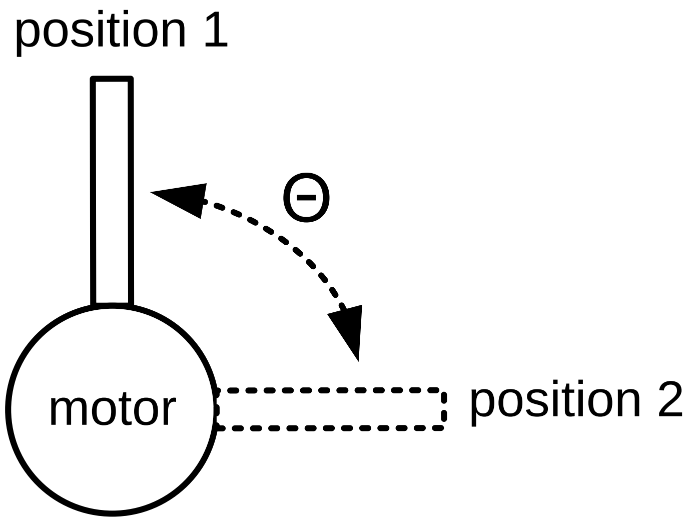
(Chapter 1.5)

Adaptive cruise control

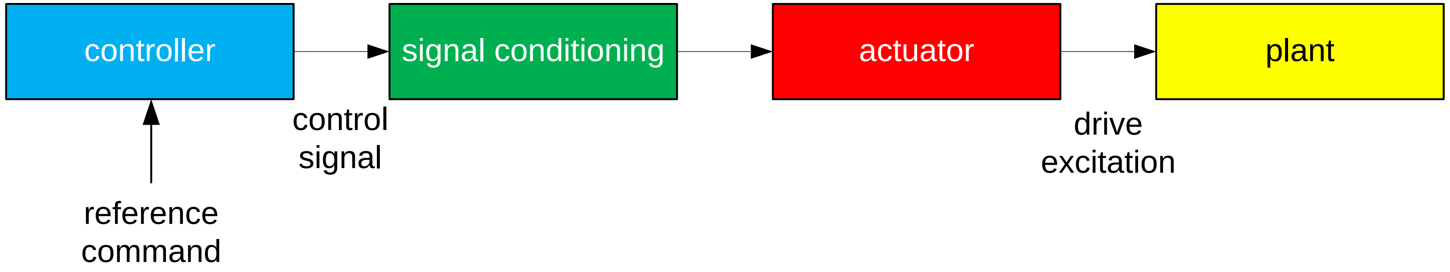


Example – position control

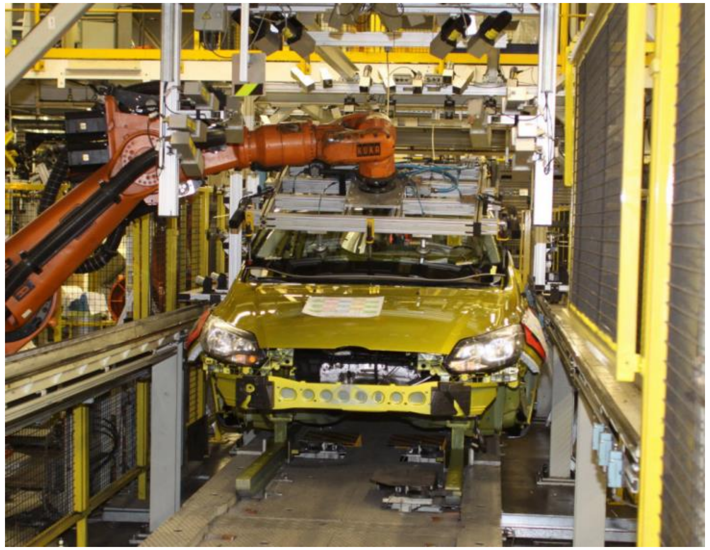


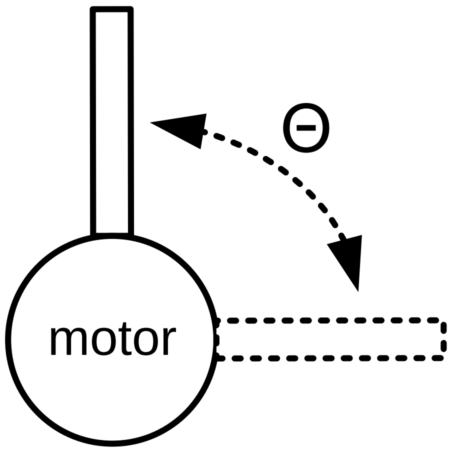


- robot arm needs to travel fixed distance (angle)
- starts at known location, ends at known location
- motor turns with constant speed
- use open-loop controller

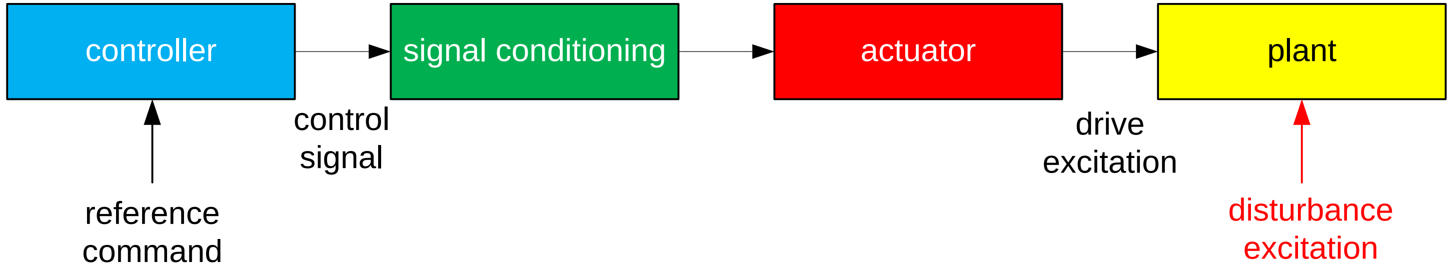


Example – position control

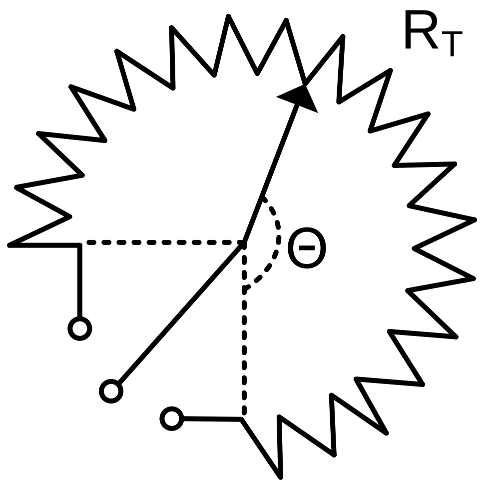


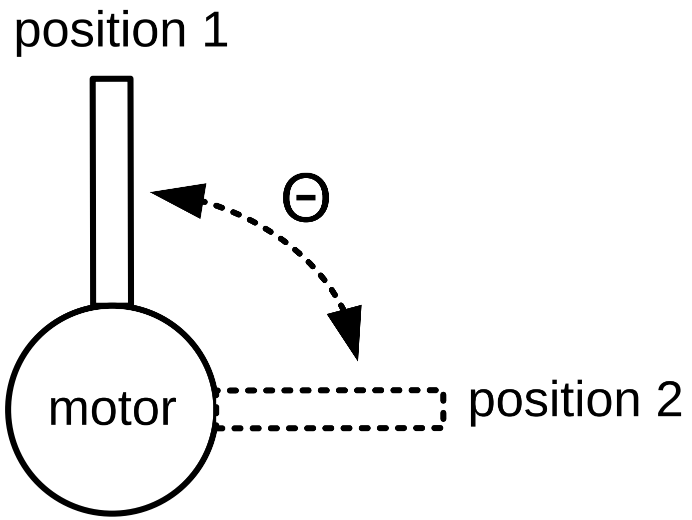


- external disturbances
  - motor speed not constant
  - start, end location not exact
- measure position of arm and use in control system

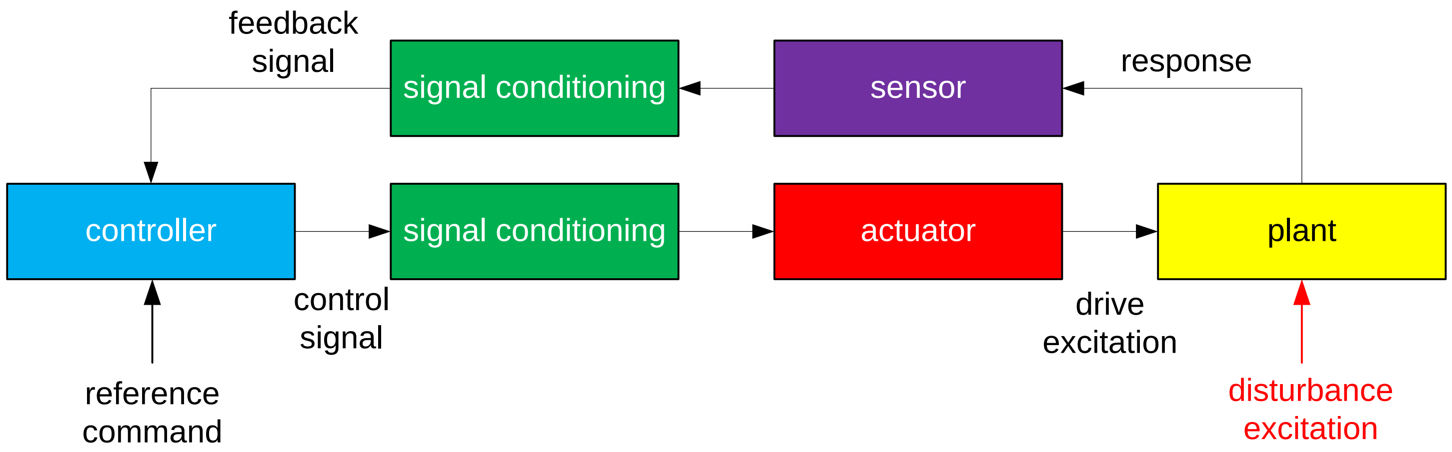


Example – position control

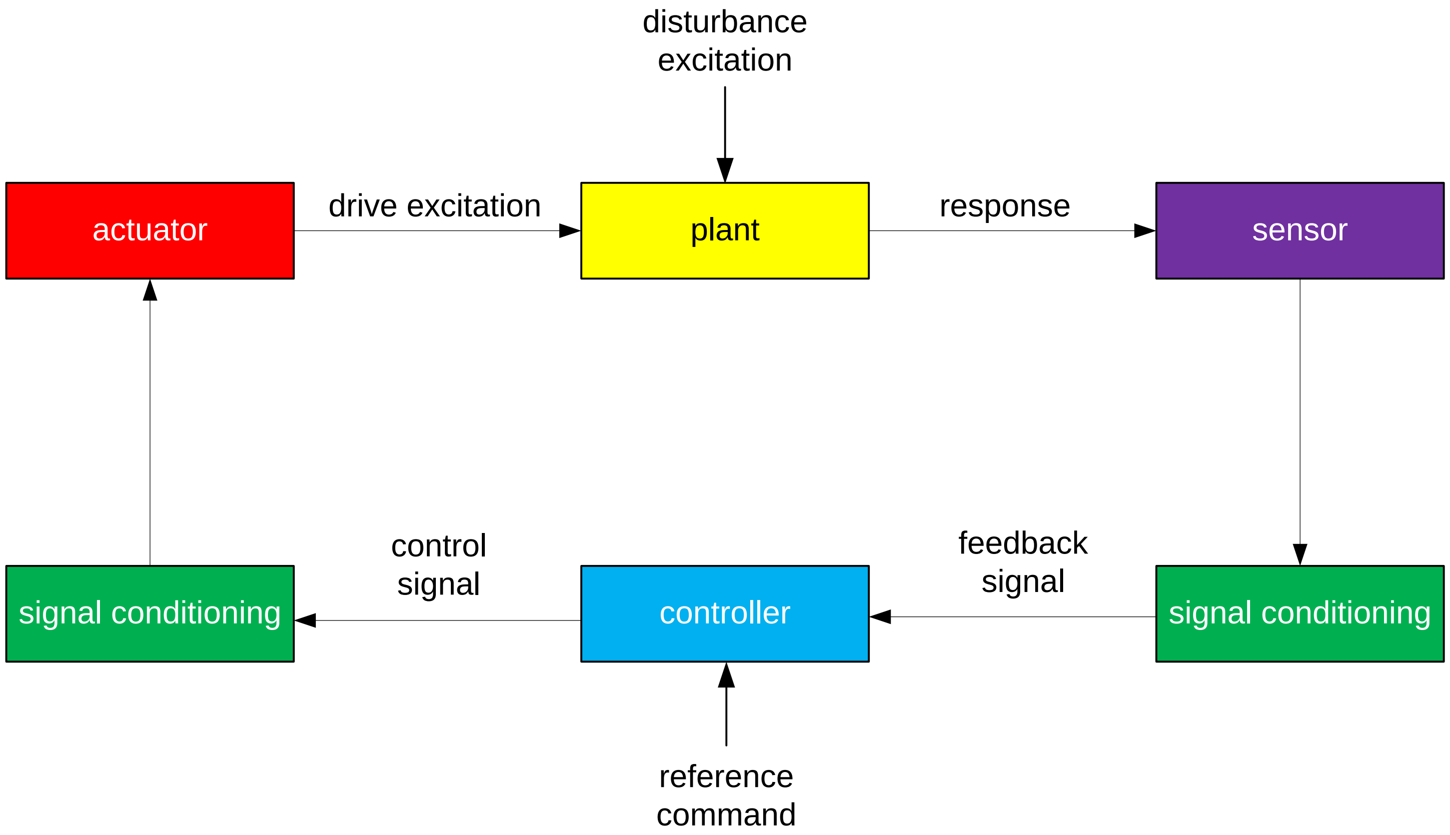




- use potentiometer to measure position
- input: desired angle of arm, output: engine on/off



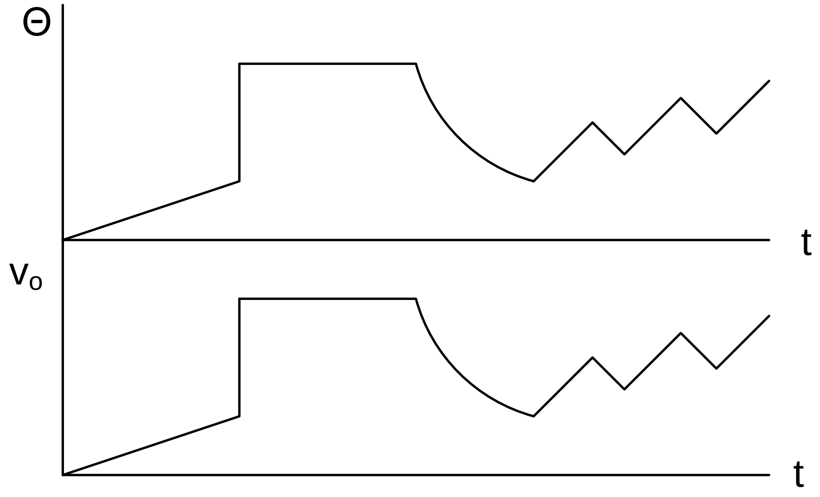
Feedback (closed-loop) control

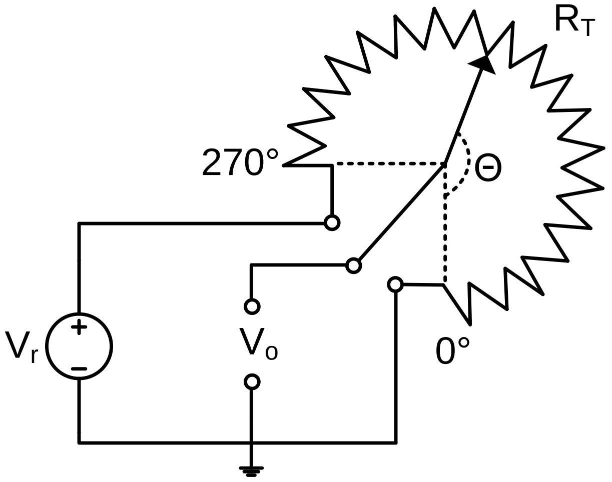


- measure response and compare with reference to minimize error
- several common feedback control strategies exist
  - on-off control
  - proportional (P) control
  - proportional control with integral (I) and derivative (D) action

Example – position control

- output voltage sensor follows robot arm immediately





$$v_o(t) = \frac{\Theta(t)}{270°} V_r$$

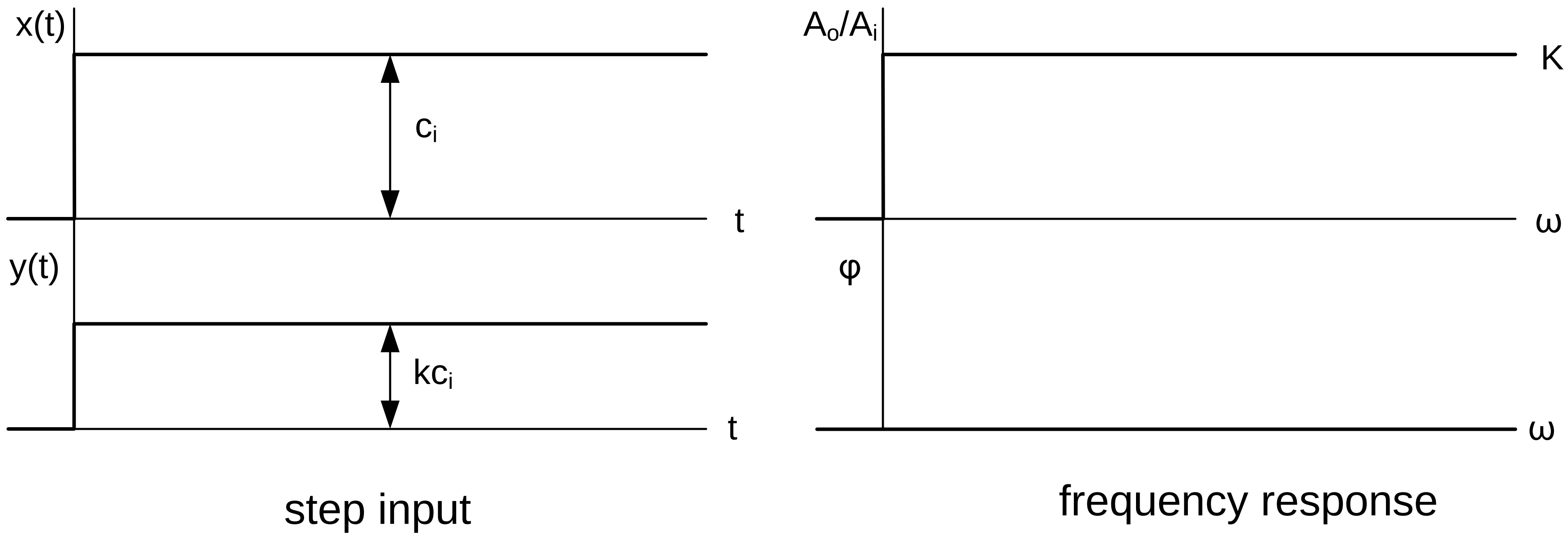
- time-dependent transfer function of robot arm

$$a_0 y(t) = b_0 x(t) \;\Rightarrow\; y(t) = \frac{b_0}{a_0} x(t) = k \cdot x(t)$$

  - static sensitivity given by k
  - system has ideal or perfect dynamic performance

Zero-order system

- zero-order system represents ideal or perfect dynamic performance
- demonstrated with response to step at input



- no dynamic error present in zero-order systems
- none of the elements in the sensor stores energy

Vehicle climate control system

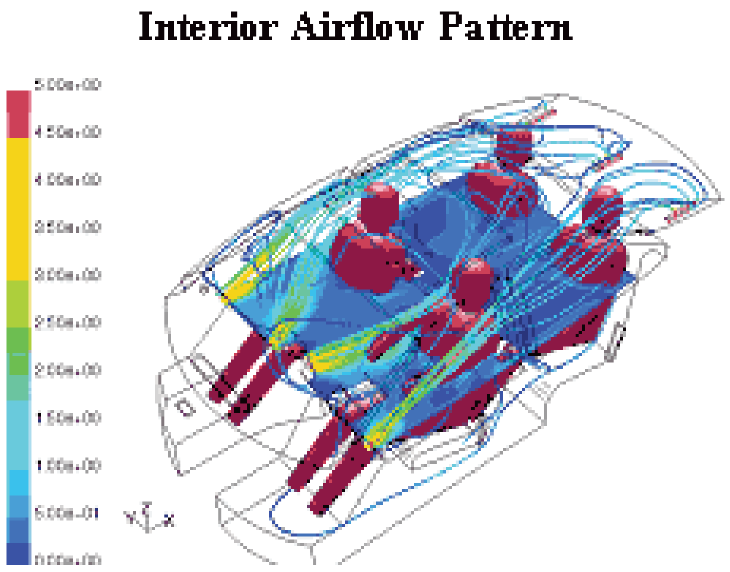
air vent (blower)

active in-car temperature

temperature set point

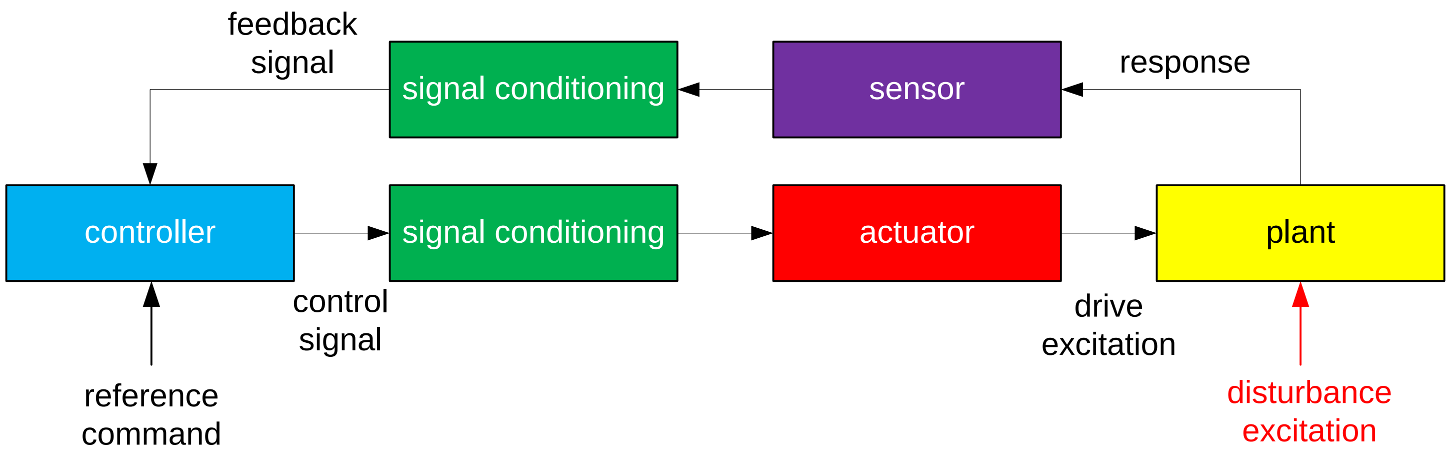
outside air temperature

intake air temperature

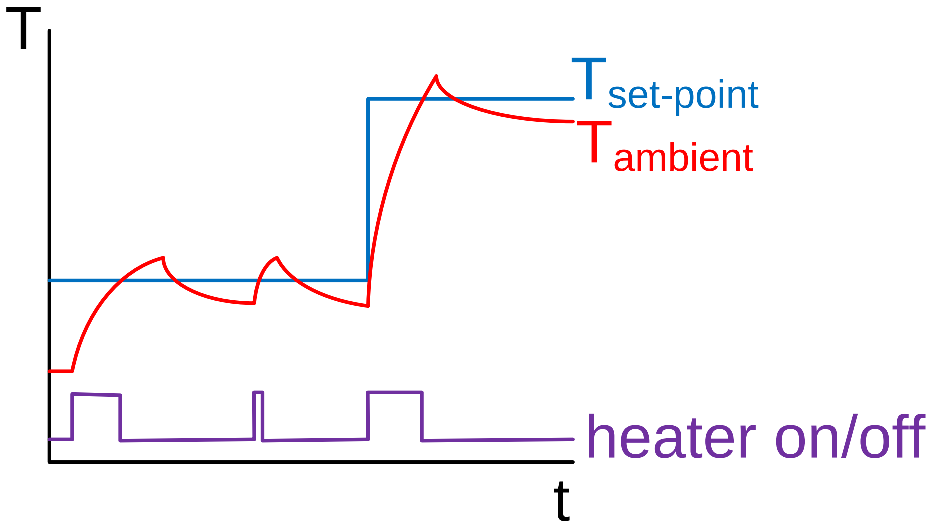


Vehicle climate control system

- on/off controller to regulate temperature

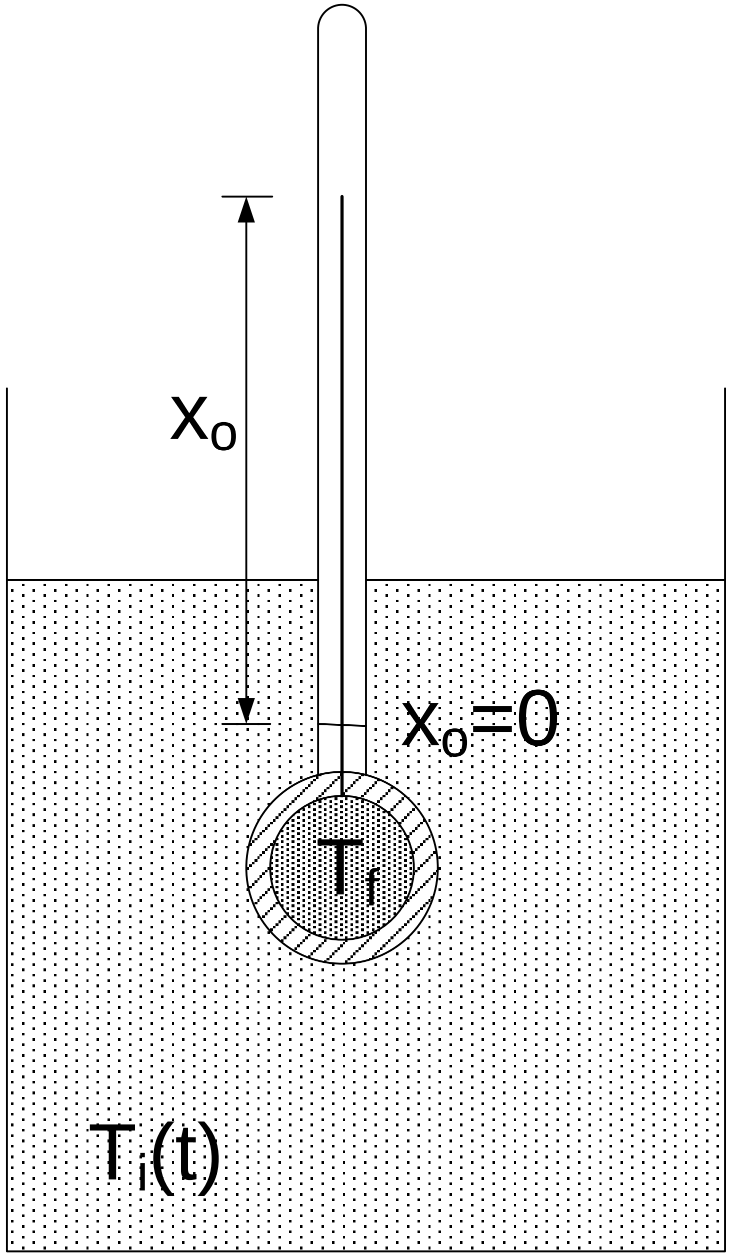


- response is not instantaneous
- thermal mass causes delay
- consider time delay in controller



Example – temperature measurement

- liquid-in-glass thermometer
  - input – temperature $T_i(t)$ of environment
  - output – displacement x_o of the thermometer fluid
  - liquid column has inertia (i.e. transfer function is not ideal)



First-order system

- first-order system contains one energy storing element
- differential equation for first-order system

$$a_1 \frac{dy(t)}{dt} + a_0 y(t) = b_0 x(t)$$

  - engineering practice to only consider x(t) and not its derivatives
  - solve equation to obtain transfer function

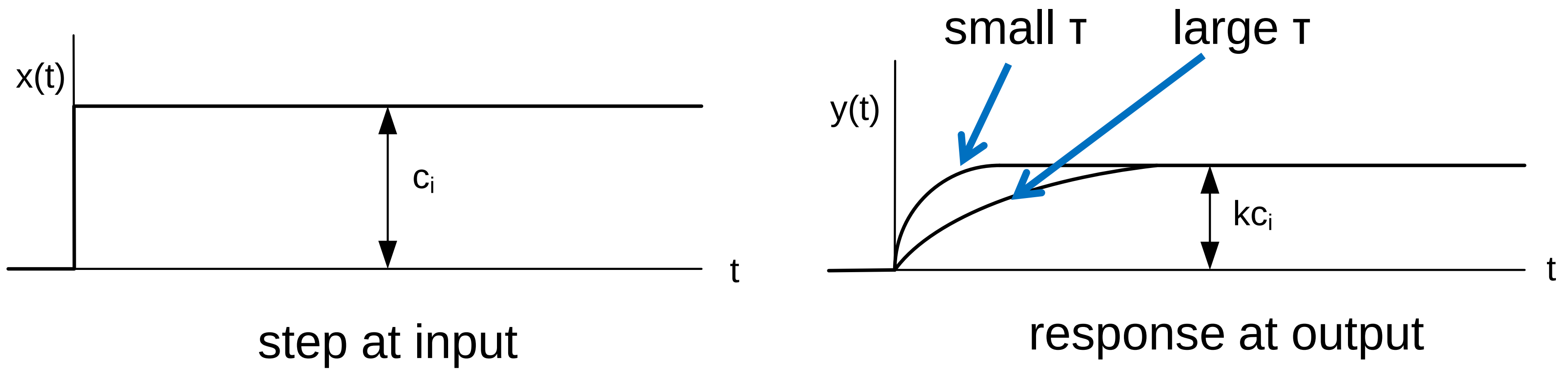
$$\left.\begin{array}{r} \dfrac{a_1}{a_0}\dfrac{dy(t)}{dt} + y(t) = \dfrac{b_0}{a_0} x(t) \\ k = \dfrac{b_0}{a_0},\ \tau = \dfrac{a_1}{a_0} \end{array}\right\} \Rightarrow \quad (\tau s + 1)Y(s) = k \cdot X(s) \Rightarrow \quad \frac{Y(s)}{X(s)} = \frac{k}{\tau s + 1}$$

    - k – static sensitivity
    - τ – time constant

First-order system

- $\frac{Y(s)}{X(s)} = \frac{k}{\tau s + 1}$, with $k = \frac{b_0}{a_0}$, $\tau = \frac{a_1}{a_0}$

- static input implies all derivatives are zero
- static sensitivity (k) is the amount of output per unit input when the input is static (constant)
- time constant (τ) determines the lag of the output signal on a change in the input signal

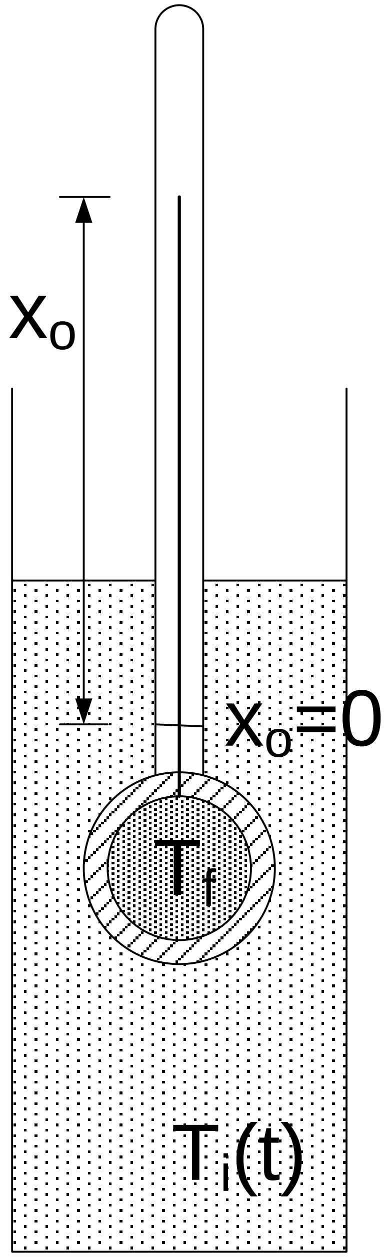


Example – liquid-in-glass thermometer

- conservation of energy provides relation between
- fluid temperature (T_f) and liquid temperature (T_i)

$$V_b \rho C \frac{dT_f}{dt} + U A_b T_f = U A_b T_i$$

  - V_b – volume of bulb [m3]
  - ρ – mass density of thermometer fluid [kg/m3]
  - C – specific heat of thermometer fluid [J/(kg°C)]
  - U – overall heat-transfer coefficient across bulb
  - wall [W/(m^2°C)]
  - A_b – heat transfer area of bulb wall [m^2]



Example – liquid-in-glass thermometer

- conservation of energy provides relation between
- fluid temperature (T_f) and liquid temperature (T_i)

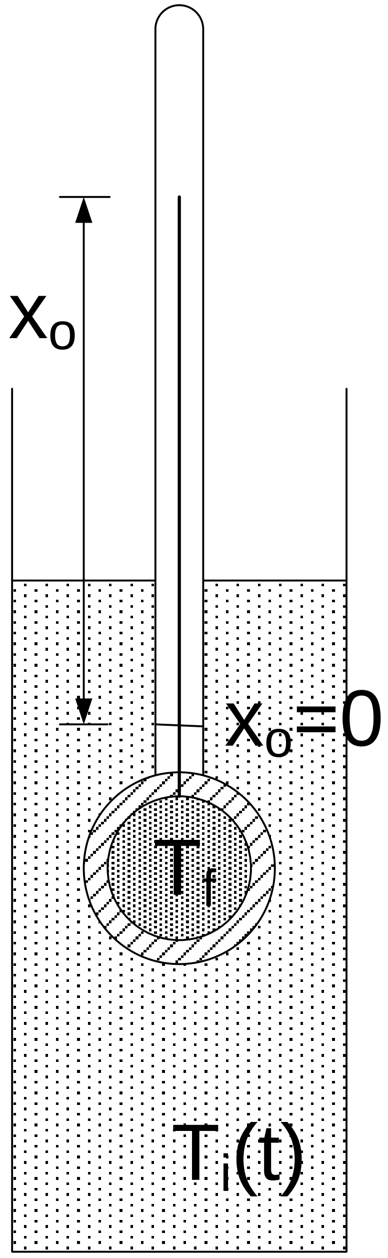
$$V_b \rho C \frac{dT_f}{dt} + UA_b T_f = UA_b T_i$$

- relation between liquid level (x_o) and fluid temperature (T_f)

$$x_o = \frac{K_{ex} V_b}{A_c} T_f$$

  - x_o – displacement from reference mark [m]
  - K_{ex} – differential expansion coefficient of fluid and bulb [$m^3/(m^3{}^\circ C)$]
  - V_b – volume of bulb [m^3]
  - A_c – cross sectional area of capillary tube [m^2]

- **what are sensitivity (k) and time constant (τ)?**



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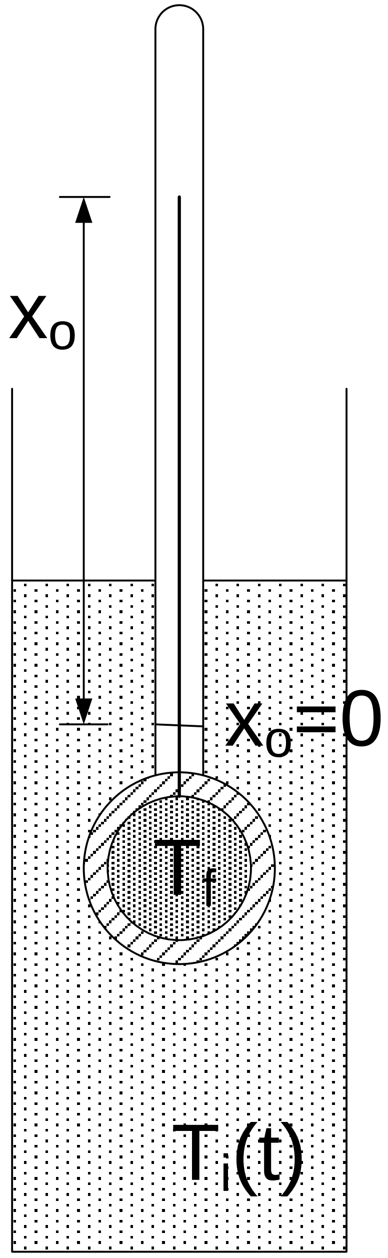
- relation between liquid level (x_o) and fluid temperature (T_f)

$$x_o = \frac{K_{ex} V_b}{A_c} T_f$$

- **what are sensitivity (k) and time constant (τ)?**
- combining equations gives differential equation
- for whole system

$$\left.\begin{aligned} V_b \rho C \frac{dT_f}{dt} + U A_b T_f &= U A_b T_i \\ x_o = \frac{K_{ex} V_b}{A_c} T_f \Leftrightarrow T_f &= \frac{A_c x_o}{K_{ex} V_b} \end{aligned}\right\} \Rightarrow \frac{\rho C A_c}{K_{ex}} \frac{dx_o}{dt} + \frac{U A_b A_c}{K_{ex} V_b} x_o = U A_b T_i$$



Example – liquid-in-glass thermometer

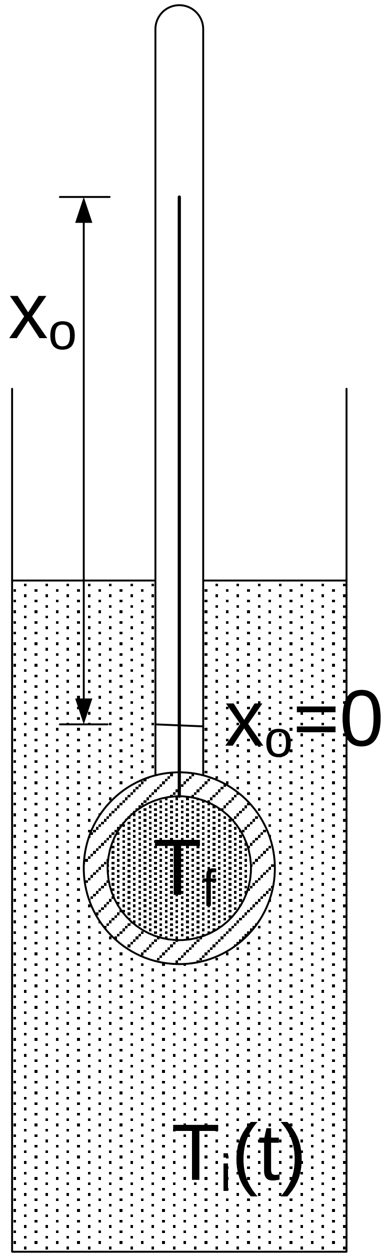
- what are sensitivity (k) and time constant (τ)?
- combining equations gives differential equation
- for whole system

$$\frac{\rho C A_c}{K_{ex}}\frac{dx_o}{dt} + \frac{U A_b A_c}{K_{ex} V_b} x_o = U A_b T_i$$

- general first-order system

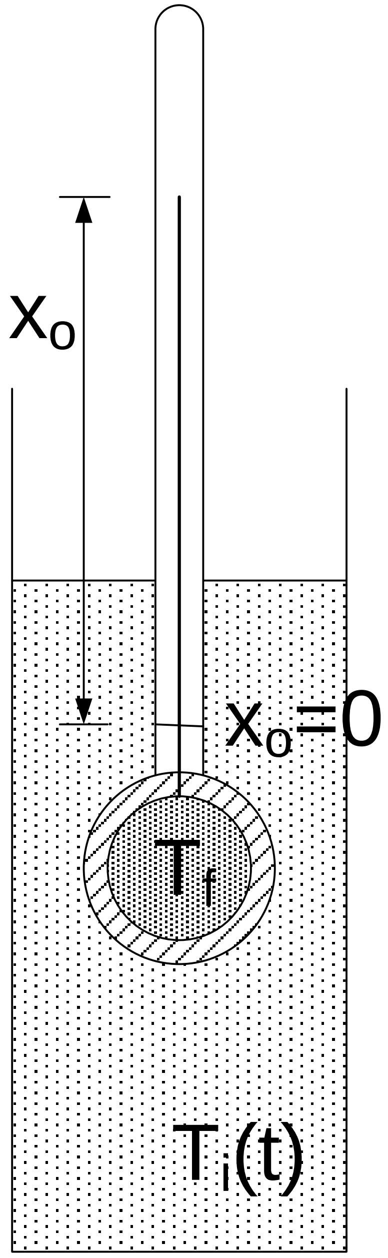
$$\frac{a_1}{a_0}\frac{dy(t)}{dt} + y(t) = \frac{b_0}{a_0} x(t) \Rightarrow \quad k = \frac{b_0}{a_0}, \ \tau = \frac{a_1}{a_0}$$

- sensitivity [m/°C] $k = \frac{K_{ex} V_b}{A_c}$
- time constant [s] $\tau = \frac{\rho C V_b}{U A_b}$



Example – liquid-in-glass thermometer

- what are sensitivity (k) and time constant (τ)?
- sensitivity [m/°C] $k = \frac{K_{ex} V_b}{A_c}$
- time constant [s] $\tau = \frac{\rho C V_b}{U A_b}$
- sensitivity and time constant related to physical parameters
- larger sensitivity (k) requires larger bulb volume (V_b)
- larger bulb volume (V_b) increases time constant (τ)
- effect partially offset by increased contact area (A_b)
- careful selection of parameters is required



First-order system

- transfer function is given in Laplace domain

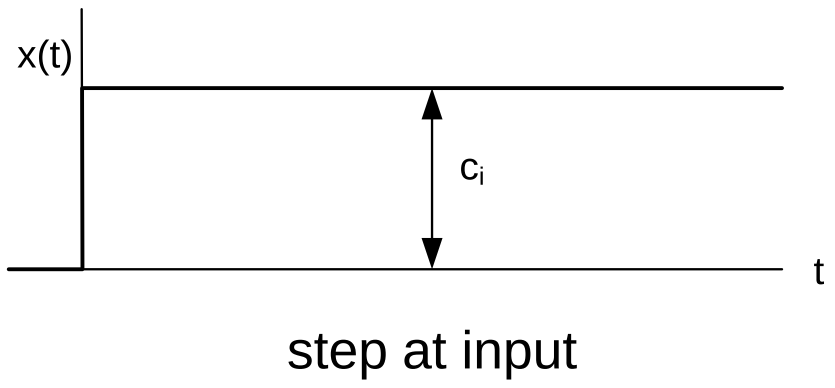
$$\frac{Y(s)}{X(s)} = \frac{k}{\tau s + 1}$$

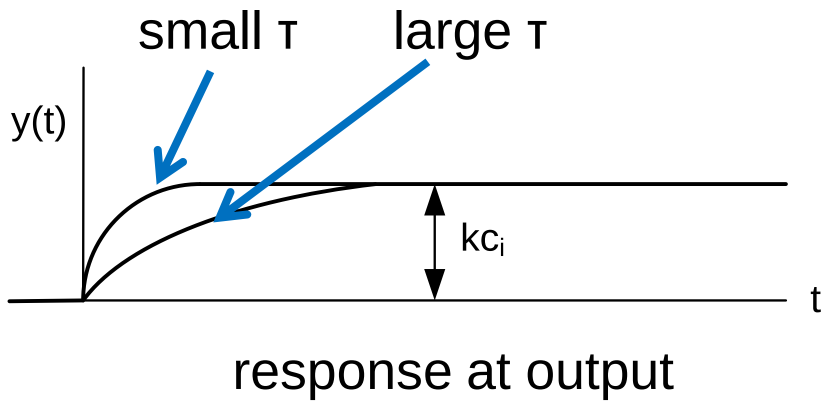
- what is the response in the time domain to an actual signal?
  - substitute X(s) with model of input signal
  - apply inverse Laplace transform

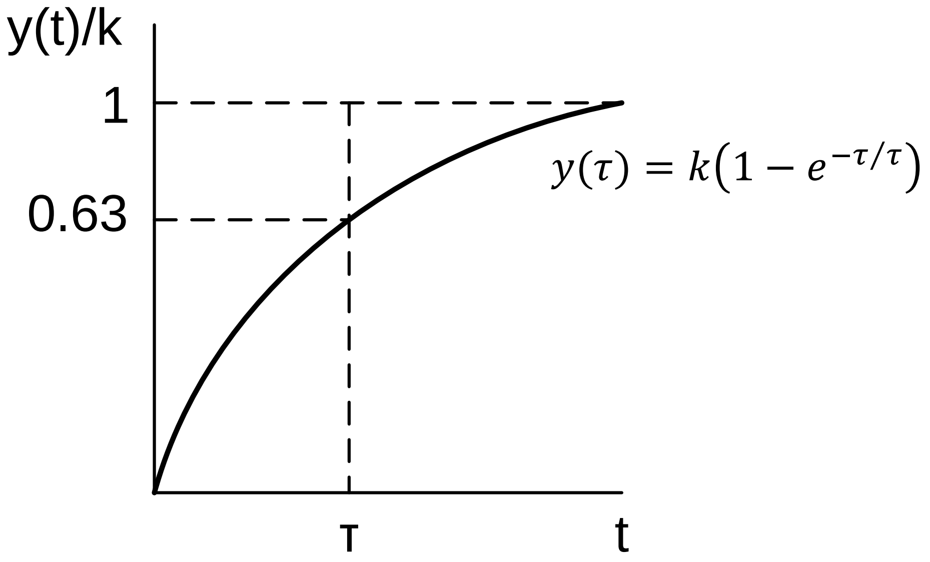
- systems usually characterized for some common test inputs
  - step
  - ramp
  - sinusoid
- common test inputs provide insight in behavior of system when real signal is applied

First-order system – response to step

- response to step input (c_i=1) is given by $y(t) = k\left(1 - e^{-t/\tau}\right)$





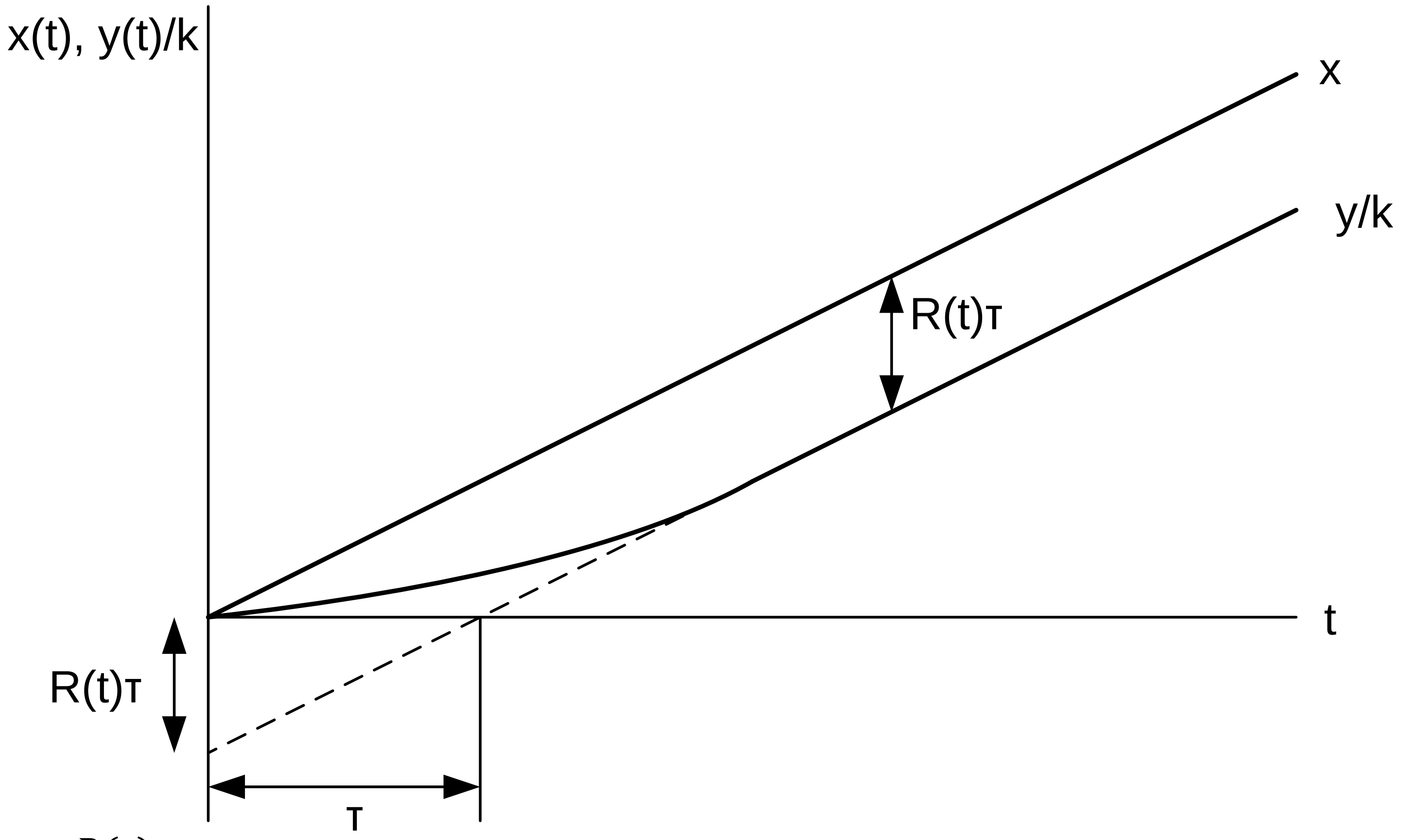


- two important characteristics
  - dynamic error $e_d = \lim_{t \to inf} y(t) - kx(t)$
  - delay $t_d = \tau$

- step response has no dynamic error
- step response has a delay

First-order system – response to ramp

- response to ramp R(t) input given by $y(t) = \begin{cases} \left(1+\tau e^{-t/\tau}\right)kR(t) & t<0 \\ \left(1+\tau e^{-t/\tau}-\tau\right)kR(t) & t \geq 0 \end{cases}$



- dynamic error $e_d = R(t)\tau$
- delay $t_d = \tau$

First-order system – response to sinusoid

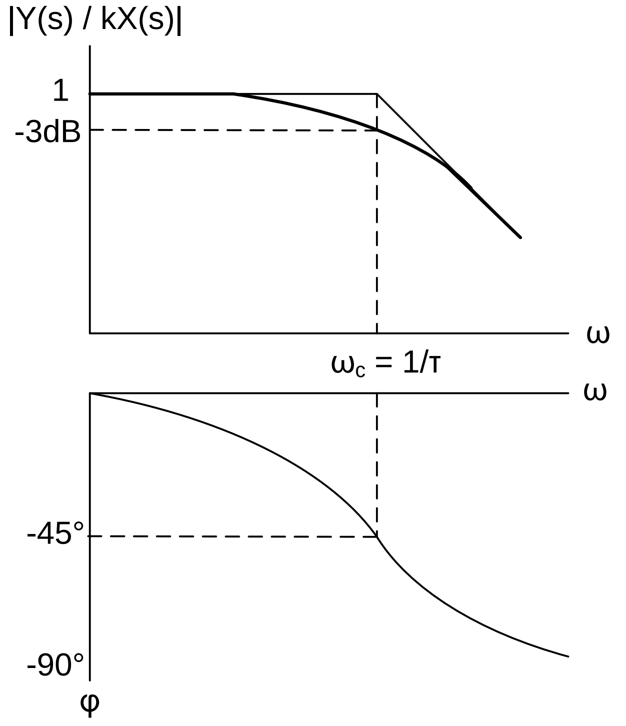
- transfer function in Laplace domain

$$\frac{Y(s)}{X(s)} = \frac{k}{\tau s + 1}$$

- response to input x(t) = A_i sin(ωt) given by y(t) = A_o sin(ωt + ϕ)

- amplitude ratio $\frac{A_o}{A_i} = \left|\frac{Y(j\omega)}{X(j\omega)}\right| = \frac{k}{\sqrt{\tau^2\omega^2 + 1}}$

- phase angle $\phi = \angle\frac{Y(j\omega)}{X(j\omega)} = \arctan(-\omega\tau)$



- "ideal" (zero-order) sensor has

$$\frac{Y(j\omega)}{X(j\omega)} = k \angle 0°$$

- approached when ωτ is small

First-order system – response to sinusoid

- response in time domain is given by

$$\frac{kA_i\tau\omega e^{-t/\tau}}{1+\omega^2\tau^2}+\frac{kA_i}{\sqrt{1+\omega^2\tau^2}}\sin(\omega t+\arctan(-\omega t))$$

- dynamic error $e_d = 1 - \frac{1}{\sqrt{1+\omega^2\tau^2}}$

- delay $t_d = \frac{\arctan(\omega\tau)}{\omega}$

First-order system – response to sum of sinusoids

- signal processing can remove
  - amplitude attenuation
  - phase shift
- if the input would be a pure sine wave...

- a more realistic signal may look like

$$x(t) = 1\sin(2t) + 0.3\sin(20t)$$

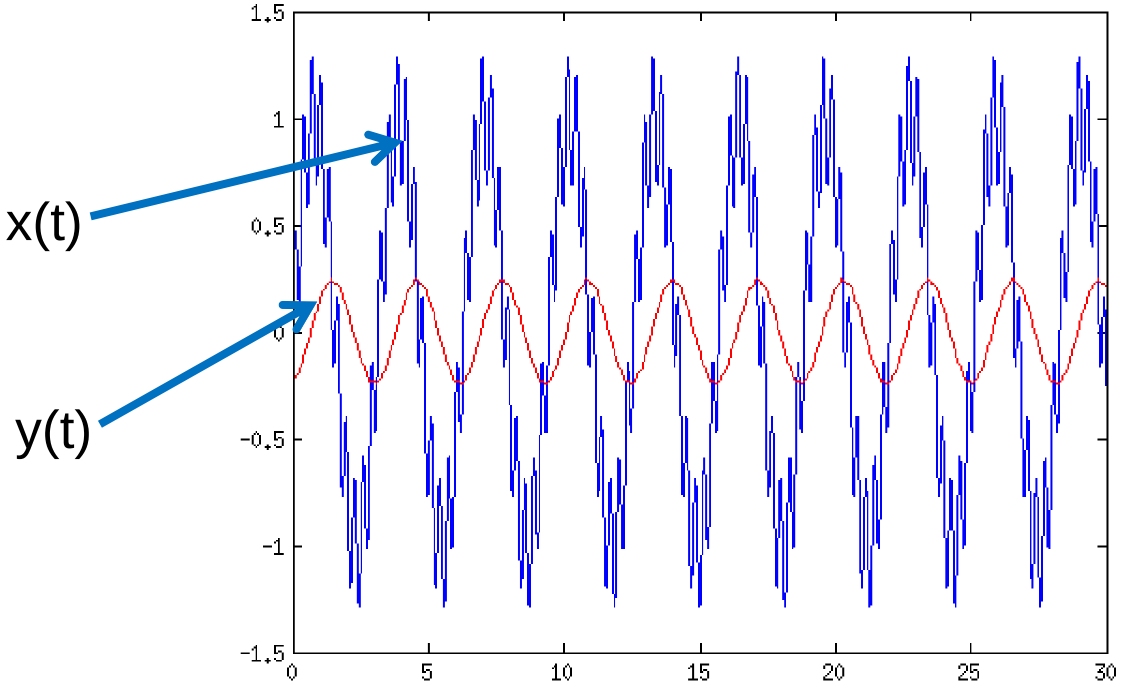
- signal is measured with first-order sensor with τ = 2s, and static sensitivity k
- **what is y(t)?**
- system is linear, therefore use superposition to find y(t)

$$\left.\frac{Y(j\omega)}{X(j\omega)}\right|_{\omega=2} = \frac{k}{\sqrt{\omega^2\tau^2+1}}\angle\arctan(-\omega\tau)\frac{k}{\sqrt{16+1}}\angle-76.0° = 0.24k\ \angle-76.0°$$

$$\left.\frac{Y(j\omega)}{X(j\omega)}\right|_{\omega=20} = \frac{0.3k}{\sqrt{1600+1}}\angle-88.6° = 0.007k\ \angle-88.6°$$

First-order system – response to sum of sinusoids

- output equal to $y(t) = 0.24k\sin(2t - 76.0°) + 0.007k\sin(20t - 88.6°)$



- observations
  - measurement of the input signal is severely distorted (high-frequency component almost invisible)
  - high-frequency component (20 rad/s) is too small compared to low frequency component

First-order system – response to sum of sinusoids

- use a different sensor with $\tau = 0.002s$

$$y(t) = 1.0k\sin(2t - 0.23°) + 0.3k\sin(20t - 2.3°)$$

- comparing output y(t) to input x(t)

$$x(t) = 1\sin(2t) + 0.3\sin(20t)$$

- observation
  - output correctly follows the input
    - amplitude almost equal (except for static sensitivity k)
    - almost no phase shift
  - selection of correct sensor parameters is very important

Example – liquid-in-glass thermometer

- time constant τ determined by immersing thermometer in a bath; it takes 28s to reach 63% of final reading
- **what is the delay when measuring the temperature of a bath that is periodically changing 2 times per minute?**
- time constant τ follows from the assignment

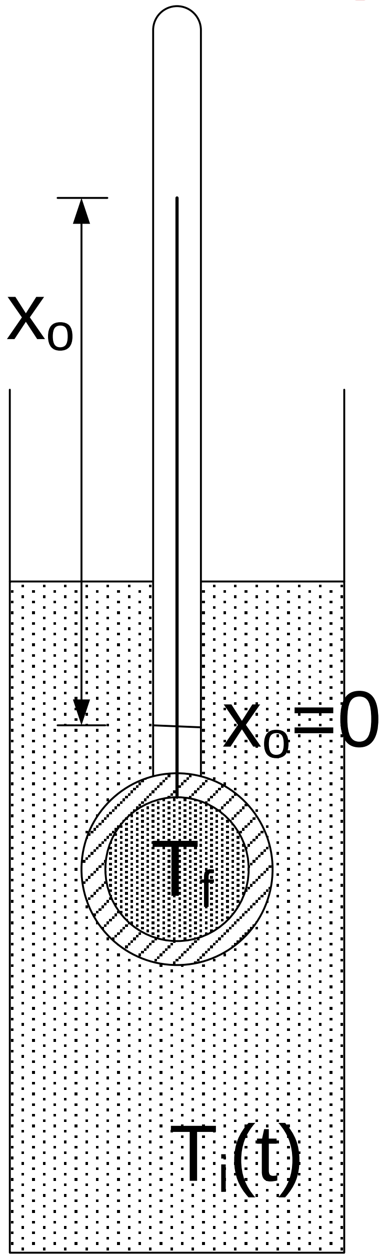
$$\tau = 28s$$

- delay when input varies cyclically is given by

$$t_d = \frac{\arctan(\omega\tau)}{\omega}$$

- angular frequency of temperature to measure

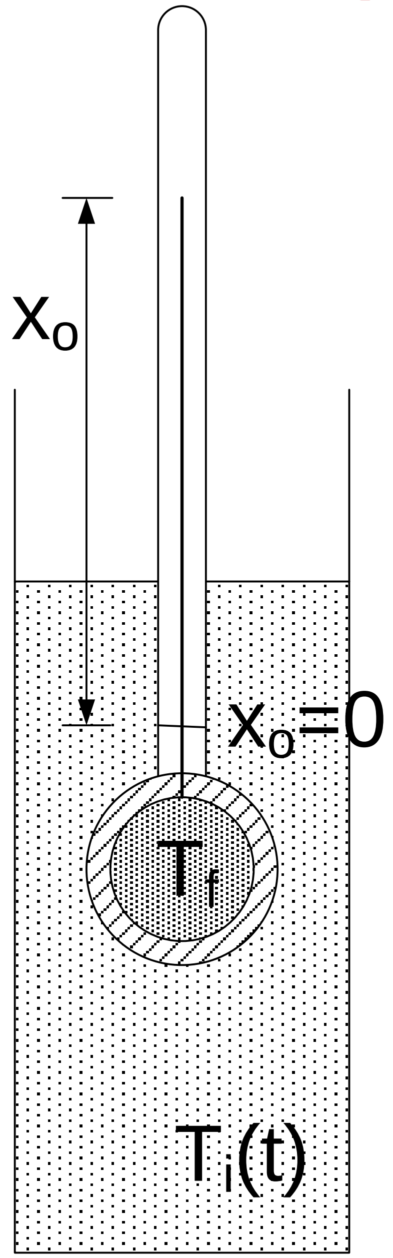
$$\omega = 2\pi \frac{2\ cycles}{60s} = 0.209 rad/s$$



Example – liquid-in-glass thermometer

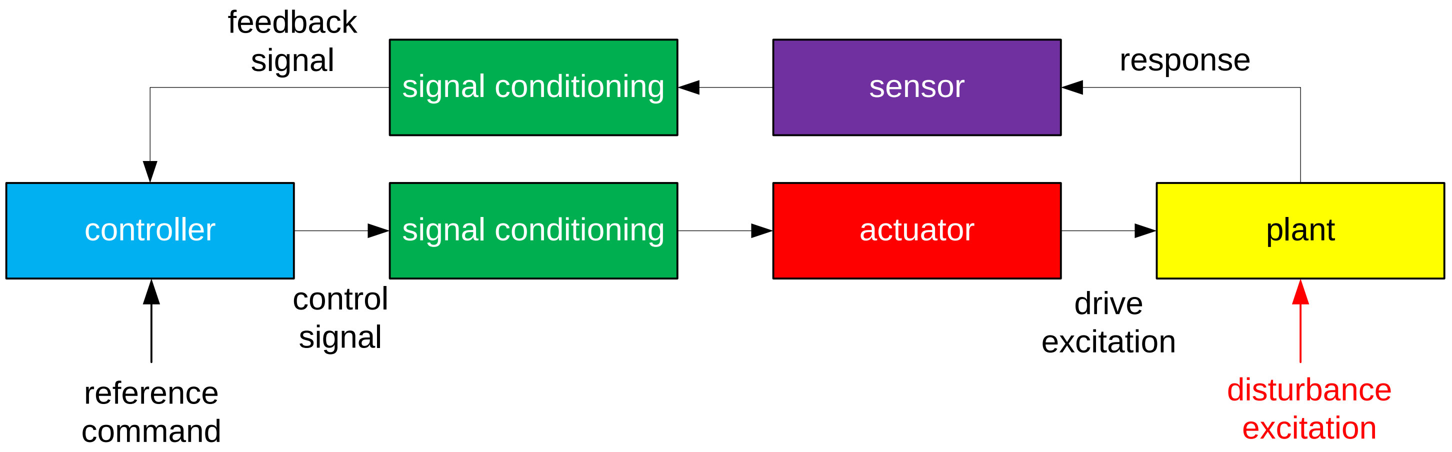
- time constant τ determined by immersing thermometer in a bath; it takes 28s to reach 63% of final reading
- **what is the delay when measuring the temperature of a bath that is periodically changing 2 times per minute?**
- delay is equal to

$$t_d = \frac{\arctan\left(\frac{0.209rad}{1s} \times 28s\right)}{0.209rad/s} = 6.7s$$

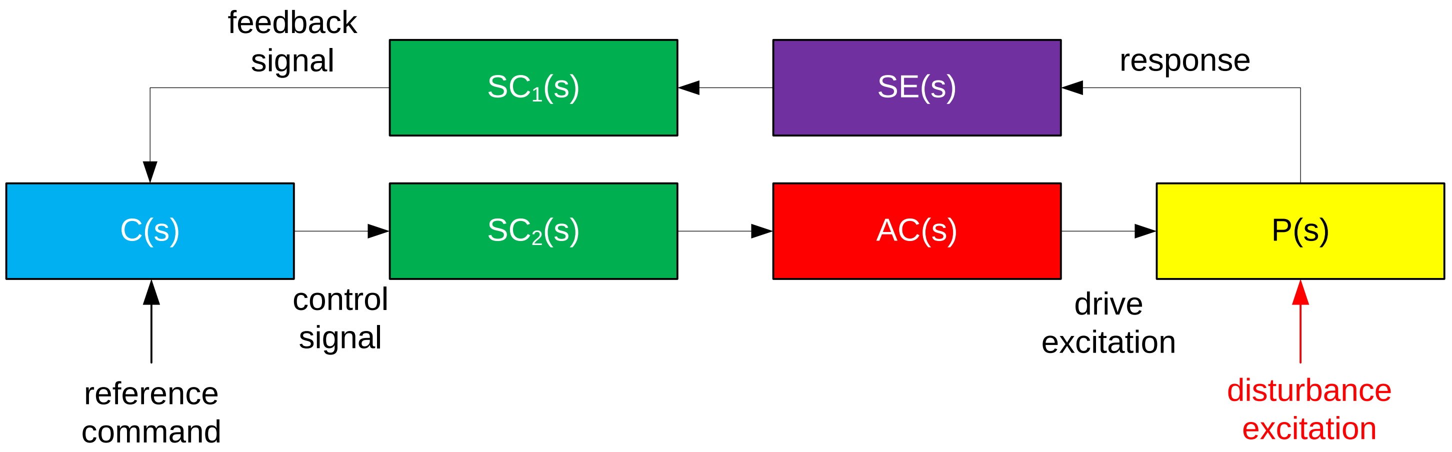


Vehicle climate control system

- feedback controller to regulate temperature

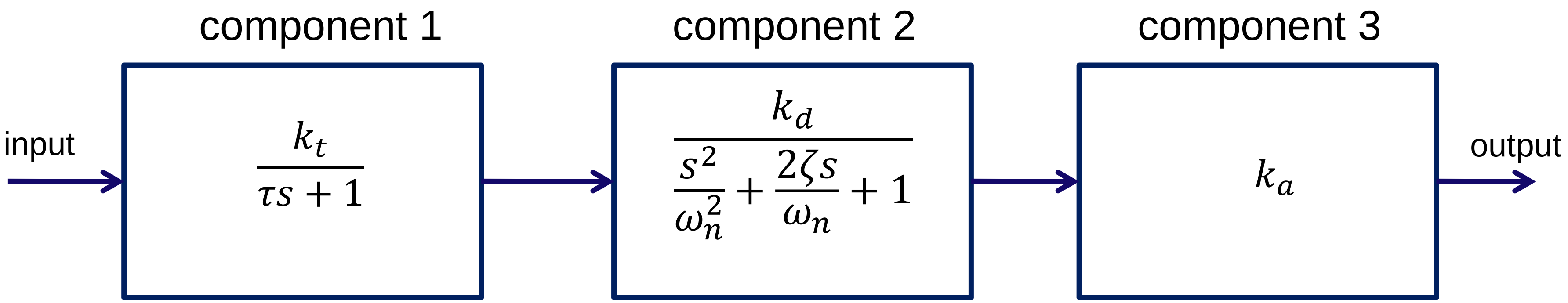


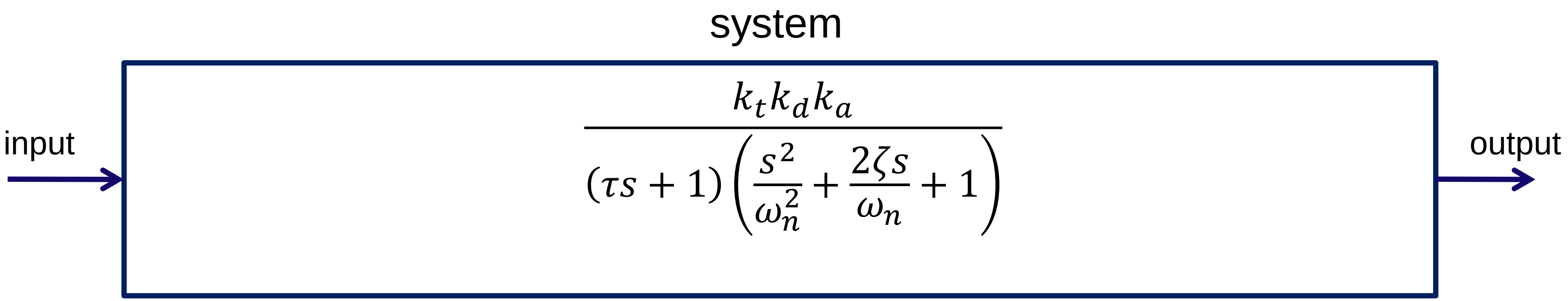
- create time-dependent model for each component



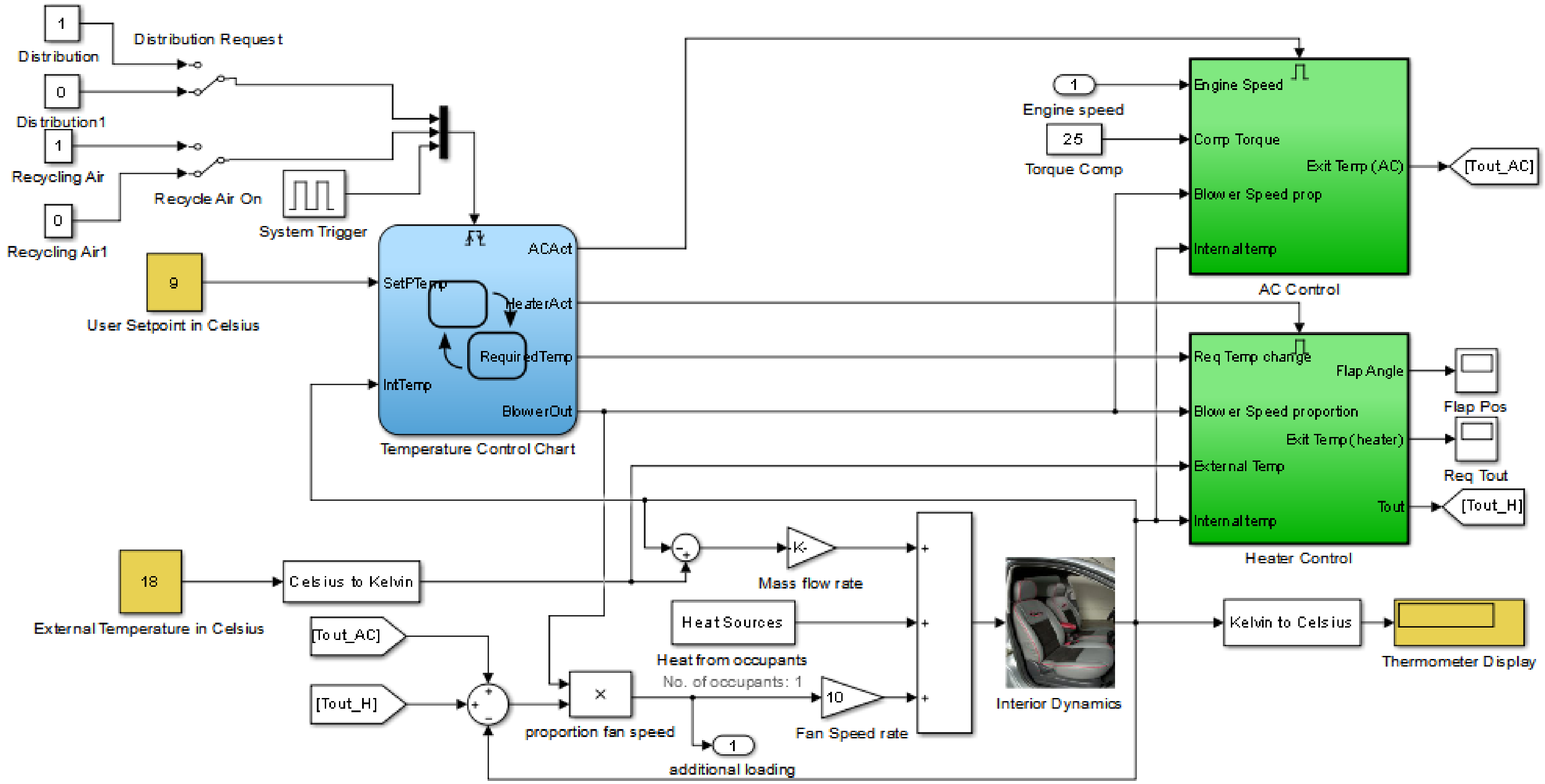
System model

- system may consist of many different components each with their own transfer function
- combination of transfer functions of all components gives transfer function of system





Vehicle climate control system



source: http://www.mathworks.nl/help/simulink/examples/vehicle-electrical-and-climate-control-systems.html