Final Exam
5AIB0 Sensing, Computing, Actuating
12-8-2014, 14.00-17.00

- This final exam consists of 3 question for which you can score at most 100 points. The final grade for this exam is determined by dividing the number of points you scored by 10.
- The solutions to the exercises should be clearly formulated and written down properly. Do not only provide the final answer. Explain your choices and show the results of intermediate steps in your computation.
- The use of a simple calculator is allowed. No graphical calculator or laptop may be used during the interim exam.

Formulae sheet

Characteristic temperature of material: $B_{T_1/T_2} = \frac{ln\left(\frac{R_2}{R_1}\right)}{\frac{1}{T_1}-\frac{1}{T_2}}$

Resistance: $R = \frac{m}{ne^2\tau}\frac{l}{A} = \rho\frac{l}{A}$

Strain: $\epsilon = \frac{dl}{l}$

Stress: $\sigma = \frac{F}{A} = E\frac{dl}{l}$

Poisson's ratio: $v = -\frac{dt/t}{dl/l}$

Change in specific resistance due to volume change (for metals): $\frac{d\rho}{\rho} = C\frac{dV}{V}$

Change in resistance due to strain: $\frac{dR}{R} = G\epsilon$

Capacitance of flat plate capacitor: $C = \frac{q}{V} = \epsilon_0\epsilon_r\frac{A}{d}$

Capacitance of cylindrical capacitor: $C = \frac{q}{V} = \epsilon_0\epsilon_r\frac{2\pi\cdot l}{ln(b/a)}$

Energy stored in capacitor: $E = \frac{C\cdot V^2}{2}$

Reluctance: $\Re = \frac{1}{\mu\mu_0}\frac{l}{A}$

Inductance: $L = \frac{N\cdot\Phi}{i} = \frac{N^2}{\Re}$

Flux: $\Phi = \mathbf{B}\times\mathbf{S}$

Magneto-motive force: $F_m = \Phi\cdot\Re = N\cdot i$

Amplitude response of Butterworth LPF: $|H(f)| = \frac{1}{\sqrt{1+\left(\frac{f}{f_n}\right)^{2n}}}$

Sideways force on electron moving through magnetic field: $\mathbf{F} = q\cdot\mathbf{v}\times\mathbf{B}$

Transverse Hall potential: $V_H = \frac{1}{N\cdot c\cdot q}\frac{i\cdot B}{d}sin(\alpha)$

Radius of warping of bimetal sensor: $r \approx \frac{2j}{3(\alpha_x-\alpha_y)(T_2-T_1)}$

Displacement of bimetal sensor: $\Delta = r(1-cos(\frac{180L}{\pi r}))$

Flow velocity and temperature difference: $v = \frac{K}{\rho}\left(\frac{e^2}{R_S}\frac{1}{T_s-T_0}\right)^{1.87}$

Voltage across P-N junction (quality factor 1): $V = \frac{kT}{q}ln\left(\frac{I}{I_0}\right)$

Saturation current through PN-junction (quality factor 1): $I_0 = BT^3e^{-E_g/kT}$

Thomson effect: $Q = I^2\cdot R - I\cdot\sigma\frac{dT}{dx}$

Peltier coefficient: $\pi_{AB}(T) = T\cdot(\alpha_A-\alpha_B) = -\pi_{BA}(T)$

Exercise 1: resistive pressure sensor **(40 points)**

Strain gauges are used amongst others to measure pressure. Figure 1 shows two strain gauges that are attached to a thin metal strip ($E = 200 \cdot 10^9$ N/m^2). The strain gauges are combined with two resistors with a fixed value into a complete bridge. When unloaded, each strain gauge has a resistance of 150 Ω. The fixed resistors also have a resistance of 150 Ω. The strain gauges have a gage factor of 1.50. To prevent damage to the strain gauges, the maximal current through them should be limited to 10 mA.

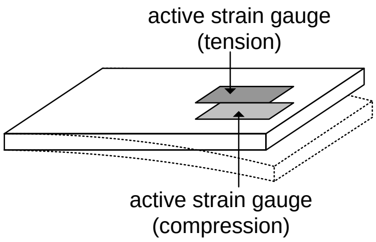

Figure 1: Metal strip with two active strain gauges.

The two strain gauges and the two fixed resistors are connected in a bridge circuit with a voltage supply V_r. The electrical equivalent circuit of this sensor is shown in Figure 2.

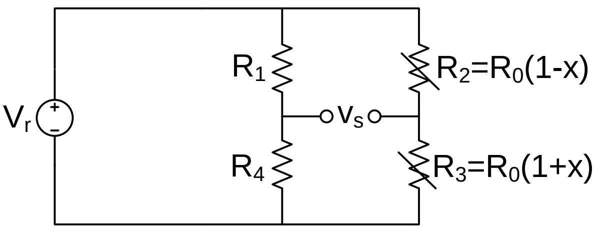

Figure 2: Bridge circuit with two strain gauges.

(3p) (a) The sensor circuit in Figure 2 contains two active strain gauges. You can also design a pressure sensor with only one strain gauge and three fixed resistors. Mention at least one advantage of the circuit shown in Figure 2 compared to a solution with only one active strain gauge.

(6p) (b) Show that the output voltage v_s of the sensor circuit is equal to:

$$v_s = -\frac{x}{2} V_r$$

(6p) (c) What value should the voltage supply V_r have to maximize the sensitivity of the sensor circuit shown in Figure 2 for a change in x?

(6p) (d) The resistance of the strain gauges, $R(x)$, does not only depend on the deformation of the strain gauges, the resistance is also influenced by the temperature of the environment. This influence of the environmental temperature on the resistance can be seen as a thermal signal y. The resistance of R_2 and R_3 is then equal to respectively $R_0(1 - x + y)$ and $R_0(1 + x + y)$. Show that the absolute error in the output voltage of the circuit, v_s, due to the temperature dependency of the strain gauges is equal to:

$$\epsilon = \left| \frac{-xy}{2(1+y)} V_r \right|$$

(Assume that the temperature dependency of the fixed resistors R_1 en R_4 can be ignored.)

Exercise continues on next page

(6p) (e) Show that the output voltage v_s of the sensor circuit shown in Figure 2 is equal to -1.13 mV when a pressure of $100 \cdot 10^6\ N/m^2$ is applied to the metal strip and $V_r = 3$ V. (Note: You may ignore the error to the the thermal dependency of the resistors R_2 and R_3 in this questions.)

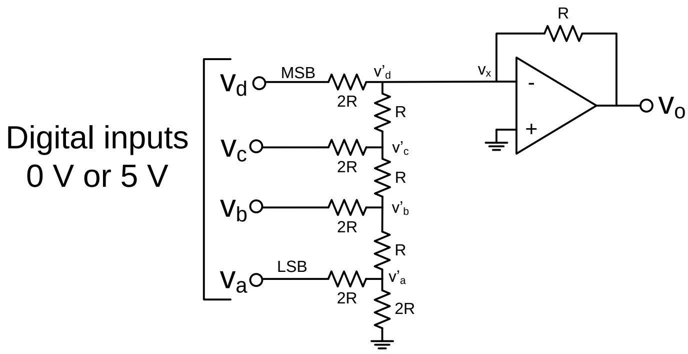

Figure 3: DA converter using ladder network.

(5p) (f) A ladder DA converter is shown in Figure 3. What is the output voltage of this DA converter when the binary input 0100 is applied to it?

(8p) (g) Give a definition (maximally 100 words) for the following terms:

- Transducer
- Sensor
- Sensitivity of a sensor
- Transfer function of a sensor

Exercise 2: Active suspension **(30 points)**

An active suspension system measures the displacement between the wheels and the car body. This can be done using a linear variable differential transformer (LVDT). The sensor uses the magnetic coupling between a primary and two secondary coils to measure the displacement of a ferromagnetic core. Figure 4 shows the electrical equivalent circuit of a LVDT sensor that is connected to a voltage supply v_1 with a frequency of 1000 Hz and an amplitude of 10 V. At this excitation frequency, a phase shift of +45° occurs between the input voltage and the output voltage of the sensors. There is no load connected to the output of the sensor.

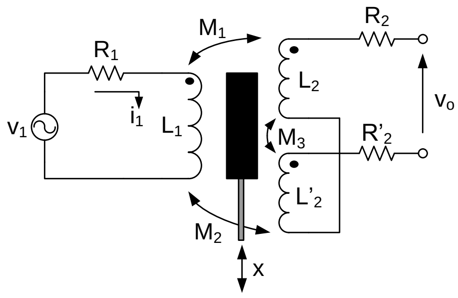

Figure 4: LVDT sensor.

(5p) (a) Show that the output voltage V_o is equal to:

$$V_o = \frac{j\omega k_x x V_1}{j\omega L_1 + R_1}$$

with $(M_2 - M_1) = k_x x$.

(5p) (b) A sinusoidal voltage supply v_1 with a frequency of 1000 Hz and an amplitude of 10 V is connected to the primary winding of the LVDT sensor shown in Figure 4. At this excitation frequency, a phase shift of +45° occurs between the input voltage and the output voltage of the sensors. The sensor has a sensitivity of 0.1 V/mm. Assume that the ferromagnetic core moves with a sinusoidal movement of 100 Hz between $x = -20$ mm and $x = +20$ mm. Draw the excititation voltage on the primary winding ($v_1(t)$), the displacement of the ferromagnetic core and the output voltage ($v_o(t)$) of the sensor. (Clearly show the dimension and units on all axis.)

(5p) (c) The magnitude and direction of the displacement can be recovered from the output voltage V_o using a phase sensitive detector. Draw the block diagram with the main components (amplifier, multiplier, low-pass filter, sensor, reference voltage).

(5p) (d) The phase shift ϕ between the input voltage V_1 and the output voltage V_o is equal to:

$$\phi = 90^\circ - arctan\left(\frac{2\pi \cdot f \cdot L_1}{R_1}\right)$$

Assume that the inductance of the primary winding of the LVDT is equal to 40 mH, i.e, $L_1 = 40$ mH. Assume further that the excitation voltage V_1 has a frequency of $f = 1000$ Hz. In this situation, a phase shift ϕ of +45° occurs between the input and output voltage of the sensor. What is the resistance of the resistor R_1?

(5p) (e) Assume that the excitation voltage V_1 has a frequency of 1600 Hz, the primary winding has a resistance R_1 of 251 Ω and an inductance L_1 of 40 mH. The frequency at which a certain phase shift occurs can be changed by adding an additional resistor R' in series or in parallel to the primary input of the sensor. What resistance should the resistor R' have such that a phase shift of +45° occurs at a frequency of 1600 Hz?

(5p) (f) An LVDT measures the displacement of an object through the change in coupling between a primary and two secondary coils. Another way to measure a displacement would be to use a Hall effect sensor. Explain the operation of a Hall effect sensor. Clearly indicate how you can use the sensor to measure a displacement.

Exercise 3: exhaust gas temperature measurement **(30 points)**

It is necessary to change the ratio between fuel and air in a combustion engine to maximize the efficiency of the engine. The objective is to keep the temperature of the gasses going through the exhaust pipe within a certain temperature range. Because of the high temperature of these gases, it is necessary to use a thermocouple to measure this temperature. A thermocouple can only measure a temperature difference. Therefore it is necessary to also add a reference sensor to the circuit. This sensor is then used to measure the absolute temperature at the reference junction.

The thermocouple in Figure 5 contains a reference junction compensation based on a RTD temperature sensor. The output voltage of this sensor (v_{ref}) is given by the following transfer function: $v_{ref} = 10mV/^\circ C \cdot T_a$ with T_a the environmental temperature at the reference junction. The output of the reference temperature sensor is connected to the reference input of the instrumentation amplifier. It holds for this instrumentation amplifier:

$$v_o - v_{ref} = \left(1 + \frac{2R_2}{R_1}\right) \frac{R_4}{R_3} (v_2 - v_1) = (1 + G)\, k\, (v_2 - v_1)$$

The ratio of the resistors in the instrumentation amplifier is equal to: $k = R_4/R_3 = 1$. The thermocouple itself is a J-type (Cn/Fe) thermocouple with a sensitivity $S_J = 54\mu$V/K.

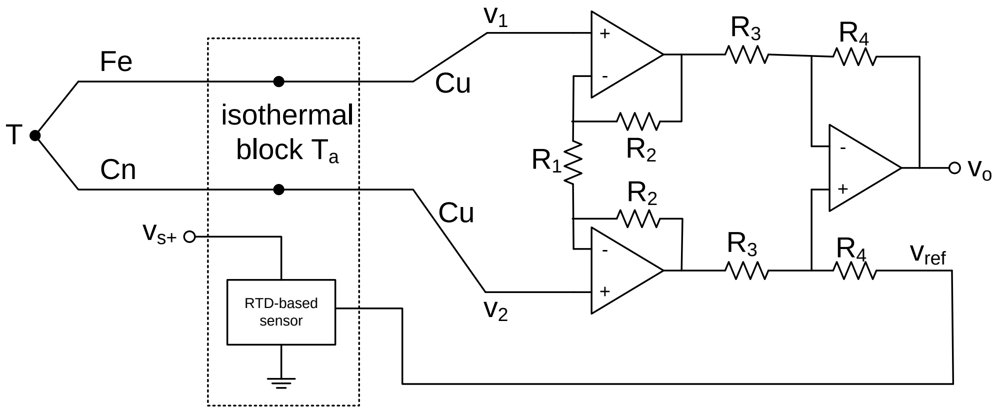

Figure 5: Compensation with a RTD-based sensor and instrumentation amplifier.

(5p) (a) There are three important law that you can use when analysing thermocouples, namely the law of the homogeneous circuits, the law of intermediate metals, and the law of intermediate temperatures. Show using these laws that the voltage across the thermocouple shown in Figure 5 is equal to:

$$v_2 - v_1 = V_T - V_{T_a} = S_J \cdot (T - T_a)$$

Hint: The Seebeck coefficient $\alpha_{Cn/Fe}$ of a J-type thermocouple is equal to: $\alpha_{Cn/Fe} = S_J$.

(5p) (b) What gain, G, should the instrumentation amplifier have to get an output voltage v_o that is independent of the environmental temperature?

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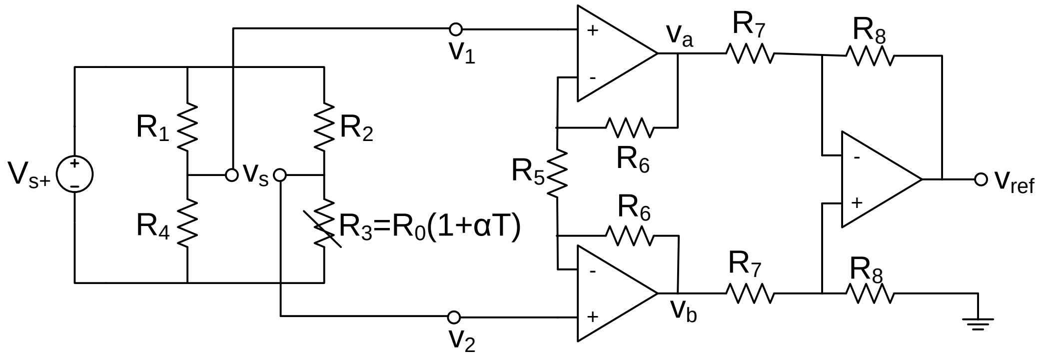

Figure 6: Implementation of "RTD-based sensor" block of Figure 5.

(5p) (c) The circuit shown in Figure 6 is used to generate the reference voltage v_{ref} for the thermocouple sensor shown in Figure 5. In other words, Figure 6 shows the internals of the block "RTD-based sensor" in Figure 5. Assume that the supply voltage v_{s+} is equal to 5 V. Assume further that resistor R_3 is a temperature dependent resistor (RTD) of type PT100. Its relation between temperature and resistance (transfer function) can be approximated with the following linear equation: $R_3(T) = R_0(1+\alpha T)$, with R_0 equal to 100Ω and $\alpha = 0.004/°C$. Assume also that $R_4 = R_5 = R_6 = R_7 = R_8 = R_0$ and $R_1 = R_2 = k \cdot R_0$. What resistance should the resistor R_2 have to ensure that the "RTD-based sensor" has a sensitivity of $10mV/°C$?

(5p) (d) The operation of a temperature dependent resistor (RTD) is based on the thermo-resistive effect. Explain briefly (maximal 200 words) how this effect works in metals.

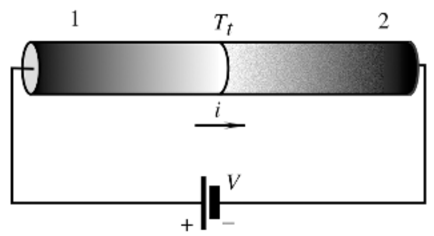

Figure 7: Peltier effect in single junction.

(5p) (e) Figure 7 shows a single junction of two different materials. Explain (in maximally 200 words) why the Peltier effect results in the production or liberation of energy at a junction when a current is passed through this junction.

(5p) (f) Give at least three reasons why we prefer transducers who produce a signal in the electrical domain over transducers that produce a signal in any of the other signal domains.