Technische Universiteit **Eindhoven** University of Technology

Student name:

Student number:

Examination cover sheet

(to be completed by the examiner)

Course name: Sensing, Computing, Actuating — Course code: 5AIB0

Date: 21-6-2015

Start time: 9:00 — End time : 12:00

Number of pages: 8

Number of questions: 3

Maximum number of points/distribution of points over questions: 100

Method of determining final grade: divide total of points by 10

Answering style: open questions

Exam inspection: make appointment via email with the responsible lecturer

Other remarks:

Instructions for students and invigilators

Permitted examination aids (to be supplied by students):

- ☐ Notebook
- ☑ Calculator
- ☐ Graphic calculator
- ☐ Lecture notes/book
- ☐ One A4 sheet of annotations
- ☐ Dictionar(y)(ies). If yes, please specify:
- ☐ Other:

Important:

- examinees are only permitted to visit the toilets under supervision
- it is not permitted to leave the examination room within 15 minutes of the start and within the final 15 minutes of the examination, unless stated otherwise
- examination scripts (fully completed examination paper, stating name, student number, etc.) must always be handed in
- the house rules must be observed during the examination
- the instructions of examiners and invigilators must be followed
- no pencil cases are permitted on desks
- examinees are not permitted to share examination aids or lend them to each other

During written examinations, the following actions will in any case be deemed to constitute fraud or attempted fraud:

- using another person's proof of identity/campus card (student identity card)
- having a mobile telephone or any other type of media-carrying device on your desk or in your clothes
- using, or attempting to use, unauthorized resources and aids, such as the internet, a mobile telephone, etc.
- using a clicker that does not belong to you
- having any paper at hand other than that provided by TU/e, unless stated otherwise
- visiting the toilet (or going outside) without permission or supervision

Final Exam
5AIB0 Sensing, Computing, Actuating
21-6-2015, 9:00-12:00

- This final exam consists of 3 questions for which you can score at most 100 points. The final grade for this exam is determined by dividing the number of points you scored by 10.
- The solutions to the exercises should be clearly formulated and written down properly. Do not only provide the final answer. Explain your choices and show the results of intermediate steps in your computation.
- The use of a simple calculator is allowed. No graphical calculator or laptop may be used during the exam.

Formulae sheet

Characteristic temperature of material: $B_{T_1/T_2} = \frac{ln\left(\frac{R_2}{R_1}\right)}{\frac{1}{T_1} - \frac{1}{T_2}}$

Resistance: $R = \frac{m}{ne^2\tau}\frac{l}{A} = \rho\frac{l}{A}$

Strain: $\epsilon = \frac{dl}{l}$

Stress: $\sigma = \frac{F}{A} = E\frac{dl}{l}$

Poisson's ratio: $v = -\frac{dt/t}{dl/l}$

Change in specific resistance due to volume change (for metals): $\frac{d\rho}{\rho} = C\frac{dV}{V}$

Change in resistance due to strain: $\frac{dR}{R} = G\epsilon$

Capacitance of flat plate capacitor: $C = \frac{q}{V} = \epsilon_0\epsilon_r\frac{A}{d}$

Capacitance of cylindrical capacitor: $C = \frac{q}{V} = \epsilon_0\epsilon_r\frac{2\pi\cdot l}{ln(b/a)}$

Energy stored in capacitor: $E = \frac{C\cdot V^2}{2}$

Reluctance: $\Re = \frac{1}{\mu\mu_0}\frac{l}{A}$

Inductance: $L = \frac{N\cdot\Phi}{i} = \frac{N^2}{\Re}$

Flux: $\Phi = \mathbf{B} \times \mathbf{S}$

Magneto-motive force: $F_m = \Phi \cdot \Re = N \cdot i$

Amplitude response of Butterworth LPF: $|H\left(f\right)| = \frac{1}{\sqrt{1+\left(\frac{f}{f_n}\right)^{2n}}}$

Sideways force on electron moving through magnetic field: $\mathbf{F} = q \cdot \mathbf{v} \times \mathbf{B}$

Transverse Hall potential: $V_H = \frac{1}{N\cdot c\cdot q}\frac{i\cdot B}{d}sin(\alpha)$

Radius of warping of bimetal sensor: $r \approx \frac{2j}{3(\alpha_x-\alpha_y)(T_2-T_1)}$

Displacement of bimetal sensor: $\Delta = r(1 - cos(\frac{180L}{\pi r}))$

Flow velocity and temperature difference: $v = \frac{K}{\rho}\left(\frac{e^2}{R_S}\frac{1}{T_s-T_0}\right)^{1.87}$

Voltage across P-N junction (quality factor 1): $V = \frac{kT}{q}ln\left(\frac{I}{I_0}\right)$

Saturation current through PN-junction (quality factor 1): $I_0 = BT^3e^{-E_g/kT}$

Thomson effect: $Q = I^2 \cdot R - I \cdot \sigma\frac{dT}{dx}$

Peltier coefficient: $\pi_{AB}(T) = T \cdot (\alpha_A - \alpha_B) = -\pi_{BA}(T)$

Exercise 1: thermocouple **(30 points)**

Systems with a thermal capacity such as a thermocouple require a transfer of heat, Q, from the environment to the sensor in order to show a change in temperature. This change in energy, E, as a function of time is described by the following first-order differential equation:

$$Q = \frac{dE}{dt} = mC_V \frac{dT_s(t)}{dt} = hA_s \left(T_a(t) - T_s(t)\right)$$

, with m the weight of the sensor, C_v the specific heat of the sensor, h the heat transfer coefficient, A_s the contact surface (area) of the sensor, T_a the ambient temperature, and T_s the sensor temperature. The transfer function of such a thermocouple sensor $T_s(s)/T_a(s)$ is equal to:

$$\frac{T_s(s)}{T_a(s)} = \frac{k}{\tau s + 1}$$

, with $k = 1$ and $\tau = \frac{mC_v}{hA_s} = 3.00$ s.

(3p) (a) Because of temperature fluctuations in the environment, the ambient temperature T_a changes according to: $T_a(t) = 0.50°\text{C}\cdot sin(0.01t) + 74.99°\text{C}$. What is the steady-state output $T_s(t)$ of the sensor?

(2p) (b) Assume that the ambient temperature T_a changes according to: $T_a(t) = 30°\text{C}\cdot sin(0.001t) + 75.35°\text{C}$. Is the sensor suitable to measure these changes? (Explain your answer)

(5p) (c) Explain (in maximally 200 words) why the Peltier effect results in the production or liberation of energy at a junction of two different materials when a current is passed through this junction.

(5p) (d) The thermocouple sensor is connected in the sensing circuit as shown in Figure 1. This circuit contains a reference junction compensation based on a RTD temperature sensor. The output voltage of this sensor (v_{ref}) is given by the following transfer function: $v_{ref} = 5mV/°C \cdot T_a$ with T_a the ambient temperature at the reference junction. The output of the reference temperature sensor is connected to the reference input of the instrumentation amplifier. It holds for this instrumentation amplifier:

$$v_o - v_{ref} = \left(1 + k + \frac{R_2 + R_4}{R_g}\right)(v_2 - v_1) = G(v_2 - v_1)$$

The ratio of the resistors in the instrumentation amplifier is equal to: $k = R_4/R_3 = R_2/R_1 = 1$. The thermocouple itself is a K-type (NiCr/Ni) thermocouple with a sensitivity $S_K = 41\mu\text{V/K}$.

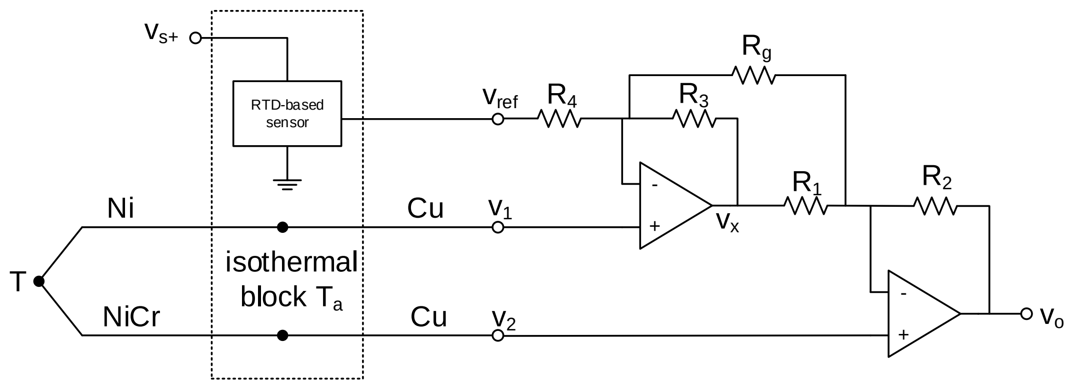

Figure 1: Compensation with a RTD-based sensor and instrumentation amplifier.

There are three important law that you can use when analysing thermocouples, namely the law of the homogeneous circuits, the law of intermediate metals, and the law of intermediate temperatures. Show using these laws that the voltage across the thermocouple shown in Figure 1 is equal to:

$$v_2 - v_1 = V_T - V_{T_a} = S_K \cdot (T - T_a)$$

Hint: The Seebeck coefficient $\alpha_{NiCr/Ni}$ of a K-type thermocouple is equal to: $\alpha_{NiCr/Ni} = S_K$.

Exercise continues on next page

(5p) (e) What gain, G, should the instrumentation amplifier have to get an output voltage v_o that is independent of the environmental temperature?

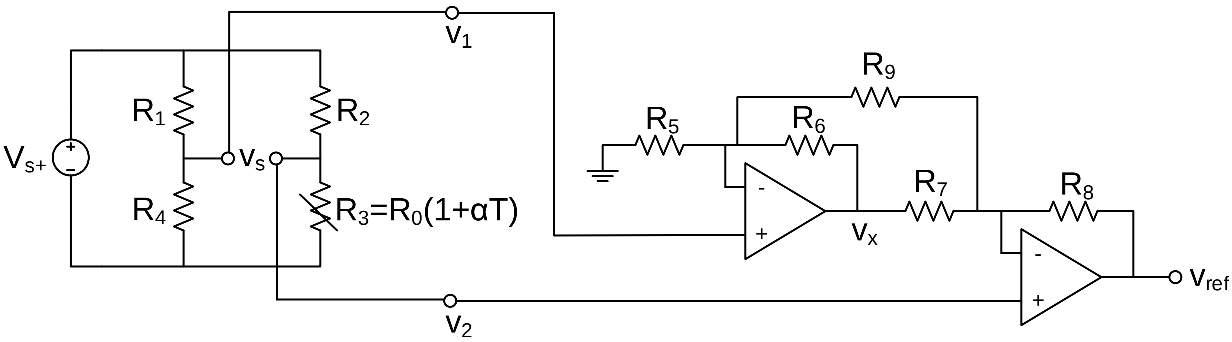

Figure 2: Implementation of "RTD-based sensor" block of Figure 1.

(5p) (f) The circuit shown in Figure 2 is used to generate the reference voltage v_{ref} for the thermocouple sensor shown in Figure 1. In other words, Figure 2 shows the internals of the block "RTD-based sensor" in Figure 1. Assume that the supply voltage v_{s+} is equal to 5 V. Assume further that resistor R_3 is a temperature dependent resistor (RTD) of type PT100. Its relation between temperature and resistance (transfer function) can be approximated with the following linear equation: $R_3(T) = R_0(1 + \alpha T)$, with R_0 equal to 100Ω and $\alpha = 0.004/°\text{C}$. Assume also that $R_4 = R_5 = R_6 = R_7 = R_8 = R_9 = R_0$ and $R_1 = R_2 = k \cdot R_0$. What resistance should the resistor R_2 have to ensure that the "RTD-based sensor" has a sensitivity of $5mV/°C$?

(5p) (g) Does the RTD-based circuit show a non-linear relation between temperature and output voltage? If so, how could these non-linearities be reduced. (Explain your answer)

Exercise 2: inductive sensor **(30 points)**

Figure 3 shows a two coil based linear displacement transformer that can be used to sense the displacement of an object over a distance x. The two coils each consist of N windings and are connected in series to each other. An excitation voltage v_e is placed over the two coils. The voltage drop over one of the coil (i.e., lower coil in Figure 3) is used as the output voltage v_o of the sensor.

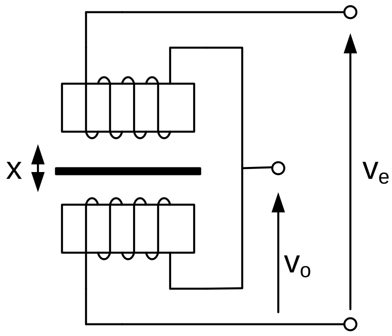

Figure 3: Inductive sensor based on two coils in series.

(5p) (a) Show that the output voltage of the sensor is equal to:

$$v_o = \frac{1-x}{2} v_e$$

(3p) (b) Is this sensor suitable to measure large displacements? (Explain your answer)

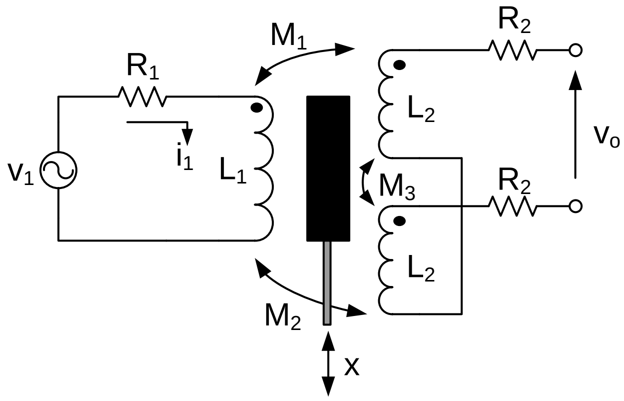

Figure 4: LVDT sensor.

(4p) (c) The linear variable differential transducer (LVDT) shown in Figure 4 is often used to sense displacements instead of the two coil sensor shown in Figure 3. When a purely resistive load R_L is connected to the output of the sensor, its output voltage is equal to:

$$V_o = \frac{j\omega k_x x R_L}{-(j\omega)^2 2L_1L_2 - j\omega(R_{2c}L_1 + 2R_1L_2) - R_1R_{2c}} V_1$$

, with $R_{2c} = 2R_2 + R_L$. For a specific LVDT holds that its primary winding has a DC resistance of 76 Ω and each of its two secondary windings have individually a DC resistance of 1600 Ω. The primary winding has an inductance of 45 mH and the two secondary windings have a combined inductance of 366 mH. How large is the phase shift between the input and output voltage when an excitation frequency of 1500 Hz is used and a load resistance of 10 $k\Omega$ is connected to the output of the sensor?

Exercise continues on next page

(5p) (d) The phase shift that you computed in the previous question can be reduced by increasing the load resistance. How large should the load resistance R_L be to ensure that there is no phase shift between the input and output voltage of the sensor when an input voltage with an excitation frequency of 1500 Hz is used?

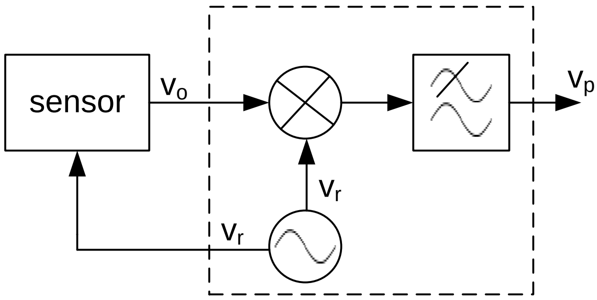

Figure 5: Sensor with phase sensitive detector.

(5p) (e) The displacement sensor from Figure 4 is connected to a phase sensitive detector. This detector consists of an analog multiplier connected to a low-pass filter. Using this processing circuit, it is possible to recover the magnitude and direction of the displacement from the output signal of the sensor. The block diagram of a phase sensitive detector is shown in Figure 5. The displacement of the object is given as a function $x(t)$. Assume further that the reference voltage v_r is equal to:

$$v_r(t) = V_r cos\left(\omega_r \cdot t\right)$$

The output voltage v_o of the sensor circuit is then equal to:

$$v_o(t) = S \cdot x(t) \cdot v_r(t)$$

, with S the sensitivity of the sensor.

Show that the output voltage of the phase sensitive detector, v_p, is equal to:

$$v_p(t) = S \cdot x(t) \cdot \frac{V_r^2}{2}$$

Hint: $cos(A)cos(B) = \frac{1}{2}\left(cos\left(A+B\right) + cos\left(A-B\right)\right)$.

(4p) (f) The output of the phase sensitive detector can be connected to an AD converter. Alternatively the output of the sensor could immediately be connected to an AD converter. Give at least one reason why we prefer to use a phase sensitive detector in front of the AD converter instead of directly connecting the output of the sensor to the AD converter.

(4p) (g) A LVDT measures the displacement of an object through the change in coupling between a primary and two secondary coils. Another way to measure a displacement would be to use a Hall effect sensor. Explain the operation of a Hall effect sensor. Clearly indicate how you can use the sensor to measure a displacement.

Exercise 3: resistive sensor **(40 points)**

A resistive temperature detector (RTD) can be used to measure the temperature of an object. Figure 6 shows a bridge circuit with an RTD which is exposed to a temperature T. This temperature will be in the range $[0°C, 100°C]$. The RTD is a PT100 sensor with $R_0 = 100\ \Omega$ and $\alpha = 0.004/°\text{C}$ at 0°C.

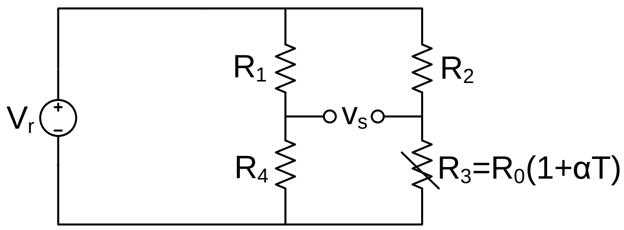

Figure 6: Bridge circuit with a RTD temperature sensor.

(5p) (a) Show that output voltage v_s of the sensor circuit is equal to:

$$v_s = -\frac{k\alpha T}{(k+1)(k+1+\alpha T)}V_r$$

, with $k = R_1/R_4 = R_2/R_0$.

(5p) (b) What ratio k should the resistors R_2/R_0 have to ensure that the non-linearity error is less then 0.8% of the reading while maximizing the sensitivity?

(7p) (c) Assume that $k = 48.6$. Assume further that the dissipation constant of the environment $\delta = 7$ mW/K. What value should the supply voltage V_r have to keep the self-heating below 0.02% of the full-scale output (FSO)?

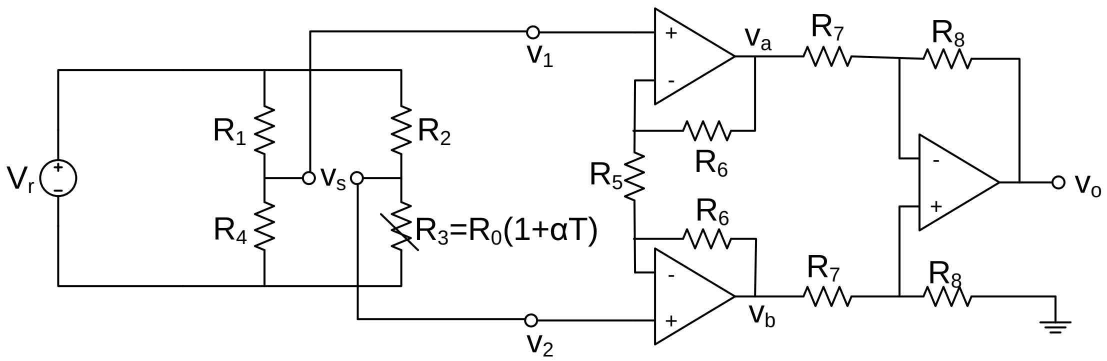

Figure 7: Sensor circuit connected to instrumentation amplifier.

(8p) (d) The sensor circuit from Figure 6 is connected to an instrumentation amplifier (see Figure 7). Show that the output voltage v_o of the circuit shown in Figure 7 is equal to:

$$v_o = \left(1 + \frac{2R_6}{R_5}\right)\frac{R_8}{R_7}\frac{k\alpha T}{(k+1)(k+1+\alpha T)}V_r$$

(5p) (e) Assume that the resistors R_6, R_7, and R_8 are all equal to 100 kΩ, $k = 48.6$, and $V_r = 5$ V. What resistance should the resistor R_5 have to ensure output of the circuit is equal to 0 V at 0°C and 5 V at 100°C?

(5p) (f) Does this instrumentation amplifier suffer from a loading effect when it is connected to the sensing circuit? Explain you answer.

Exercise continues on next page

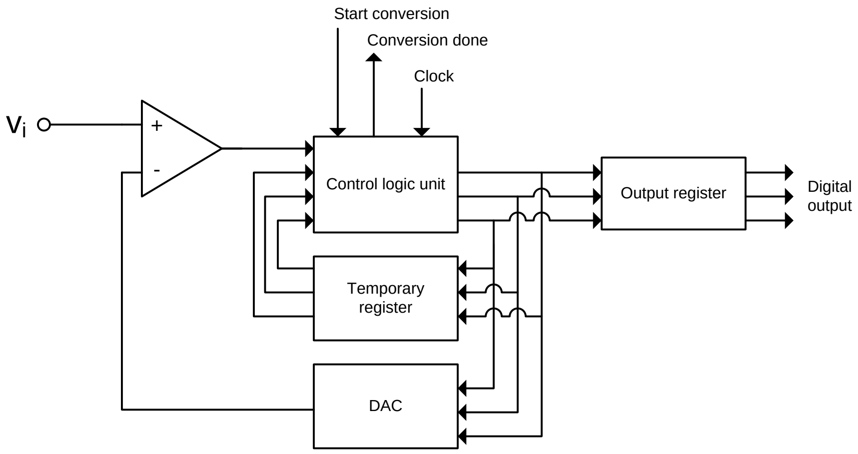

Figure 8: Successive approximation ADC.

(5p) (g) The output of the sensor circuit shown in Figure 7 is connected to the input of the successive approximation AD converter shown in Figure 8. This ADC accepts input voltages between 0 V and 5.1 V and it converts those to an 8-bit binary value. Assume that the output voltage of the sensor circuit v_o is equal to 3.00 V. Draw the DAC output (labels and levels) and its binary input for the first seven bits tested.