# Sensing, Computing, Actuating
# Lecture 3 - Systems and Control

**Exercise 1: first-order system - temperature sensor**

Systems with a thermal capacity such as a liquid thermometer or a thermocouple require a transfer of heat, $Q$, from the environment to the sensor in order to show a change in temperature. This change in energy, $E$, as a function of time is described by the following first-order differential equation:

$$Q = \frac{dE}{dt} = mC_V \frac{dT_s(t)}{dt} = hA_s \left(T_o(t) - T_s(t)\right)$$

, with $m$ the weight of the sensor, $C_v$ the specific heat of the sensor, $h$ the heat transfer coefficient, $A_s$ the contact surface (area) of the sensor, $T_o$ the environmental temperature, en $T_s$ the sensor temperature.

(a) Show that the transfer function of the sensor $T_s(s)/T_o(s)$ is equal to:

$$\frac{T_s(s)}{T_o(s)} = \frac{k}{\tau s + 1}$$

, with $k = 1$ and $\tau = \frac{mC_v}{hA_s}$.

(b) The response of the sensor to a step function on its input is given by:

$$T_s(t) = k\left(1 - e^{-t/\tau}\right)$$

Assume that the sensor has an initial temperature $T_s(0) = T_i$ when the sensor is suddenly exposed to a constant environmental temperature $T_o$. Show that the response of the sensor is equal to:

$$T_s(t) = T_o + (T_i - T_o)\, e^{-t/\tau}$$

(c) To determine the time constant $\tau$ the sensor is exposed from $t = 0$ to a (constant) environmental temperature. The temperature is measured every 3 seconds. This results in the following series of readings:

| Time (s) | 0 | 3 | 6 | 9 | 12 | 15 |
|---|---|---|---|---|---|---|
| Temperature (°C) | 20.00 | 35.54 | 39.00 | 39.78 | 39.95 | 39.99 |

What is the time constant $\tau$ from this sensor?

(d) How large is the dynamic error $\epsilon_d$ of this sensor in response to a step function? (*Hint:* $\epsilon_d = \lim_{t\to\infty} T_s(t) - k \cdot T_o(t)$ )

(e) Because of temperature fluctuations in the environment, the environmental temperature $T_o$ changes according to: $T_o(t) = 2.3°\text{C}\cdot sin(0.50t) + 39.99°\text{C}$. Assume that the time constant $\tau$ is equal to 2.00 s. What is the steady-state output of this sensor $T_s(t)$?

(f) You want to use the same sensor to measure the temperature of an object whose temperature as a function of time is given by: $T_o(t) = 2.3°\text{C}\cdot sin(20t) + 39.99°\text{C}$. Can you use this sensor to accuratly measure the fast variations in the temperature? (Explain your answer.)

**Exercise 2: second-order system - acceleration sensor**

A one-axis acceleration sensor consists of a mass whose movement can be translated into an electrical signal. This translation can be performed using for example a capacitive or piezo-electric sensor. The electrical principle is not important for this exercise, we will focus on a mechanical model of the device to analyse its operating characteristics. The figure below shows a generic model for such an acceleration sensor. The mass M is supported by a spring with a spring constant $k$ and the movement of the mass is dampened with a damper that has a damping factor $b$. The mass may only be moved along the $x$-axes with respect to the acceleration sensor body. During its use, the sensor is exposed to an acceleration $d^2y/dt^2$ and the output signal is proportional to the displacement $x_0$ of the mass $M$.

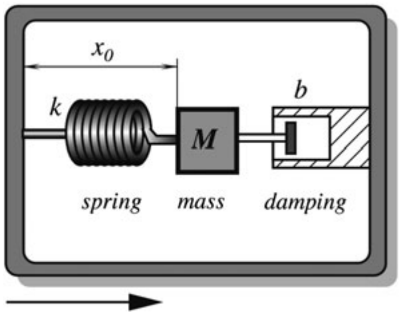


Figure 1: Mechanical model of an acceleration sensor.

(a) Show that the transfer function (in terms of the displacement of the mass $x(t)$ (output) en displacement of the sensor body $y(t)$ (input)) is equal to:

$$\frac{X(s)}{Y(s)} = \frac{M}{k}\frac{(k/M)s^2}{s^2 + (b/M)s + k/M}$$

(b) Show that the transfer function of the sensor (in terms of the acceleration $a(t)$) is equal to:

$$\frac{X(s)}{A(s)} = \frac{M}{k}\frac{(k/M)}{s^2 + (b/M)s + k/M}$$

(c) Assume that the spring constant $k$ is equal to 508.62 N/m and the mass $M$ has a weight $4.313 \times 10^{-6}$ kg. Show in a graph the relation between acceleration (x-as) and the displacement of the mass (y-as) over the range from 0 'g' till 30 'g'.

(d) Use the values of $k$ and $M$ from the previous question and assume further that the damping factor $b$ is equal to 0.047 Ns/m. You want to use the sensor to measure the displacement of an object of which its position around the centre position shows a sinusoidal movement with a frequency of 0.001 Hz. Is the sensor usable for this application? (Explain your answer.)

(e) You want to use this sensor to measure the acceleration of an object of which the accelation varies sinusoidally between -10 'g' and +10 'g' with a frequency of 0.001 Hz. Is this sensor usable for this application? (Explain your answer.)

(f) The static sensitivity of the sensor is defined as $M/k$. You can improve the static sensitivity by increasing the mass $M$. Enlarging the mass has however also an impact on the dynamic behaviour of the system. Explain how the spring constant $k$ and the damping factor $b$ should be changed to compensate the effect of the enlarged mass, while still increasing the static sensitivity of the sensor.