Sensing, Computing, Actuating
Lecture 8 - Thermistors

Exercise 1: Silicon resistive detector

The transfer function of a specific silicon-based temperature sensors for the range -60°C till +150°C is equal to:

$$R_T = R_{25}\left(\frac{273.15K + T}{298.15K}\right)^{2.3}$$

, with T the temperature in °C.

(a) What is the temperature coefficient of resistance (TCR) of this sensor at 25°C?

Answer: TCR is defined as:

$$\frac{dR_T/dT}{R_T}$$

It holds:

$$\frac{dR_T}{dT} = 2.3\frac{1}{298.15K}R_{25}\left(\frac{273.15K + T}{298.15K}\right)^{1.3}$$

At 25°C holds:

$$\left.\frac{dR_T}{dT}\right|_{T=25^\circ C} = \frac{2.3}{298.15K}R_{25}\left(\frac{298.15K}{298.15K}\right)^{1.3} = (0.0077R_{25})/K$$

Therefore it holds:

$$TCR(25^\circ C) = \left.\frac{dR_T/dT}{R_T}\right|_{T=25^\circ C} = \frac{dR/dT}{R_{25}} \cdot 100\% = 0.77\%/K$$

(b) The silicon-based temperature sensor has a non-linear response (in terms of the resistance R_T). A resistor is connected in parallel to the sensor to linearise the response (see Figure 1). Assume that $R_{25} = 1000\ \Omega$ holds for the sensor. What value should R_1 have to ensure that the transfer function of the circuit does not show any error at both ends of the range of the sensor?

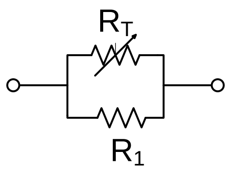


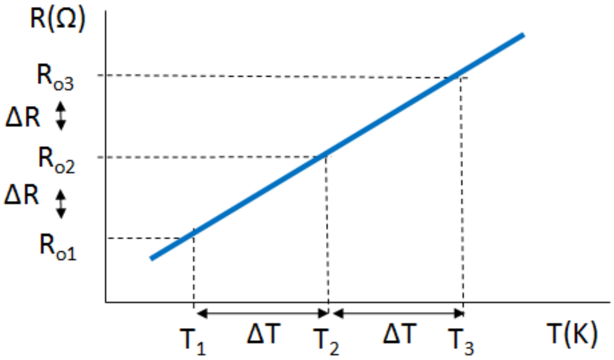
Figure 1: Linearisation of a silicon resistive detector.

Answer:

The replacement resistance of two resistors is equal to:

$$R_o = \frac{R_1 R_T}{R_1 + R_T}$$

The goal is to choose R_1 such that R_o forms a straight line through the two end points and the middle of the range. The sensor has a PTC characteristic. Hence, the linear approximation will look as follows:



(This linear approximation is exactly opposite to the version shown during the lecture since the sensor used in the lecture showed an NTC behavior.)

It should now hold:

$$T_1 - T_2 = T_2 - T_3$$

$$R_{o1} - R_{o2} = R_{o2} - R_{o3}$$

This implies:

$$\frac{R_1 R_{T1}}{R_1 + R_{T1}} - \frac{R_1 R_{T2}}{R_1 + R_{T2}} = \frac{R_1 R_{T2}}{R_1 + R_{T2}} - \frac{R_1 R_{T3}}{R_1 + R_{T3}}$$

$$\Rightarrow R_1 = \frac{R_{T2}(R_{T1} + R_{T3}) - 2R_{T1}R_{T3}}{R_{T1} + R_{T3} - 2R_{T2}}$$

It holds:

$$T1 = -60^\circ C \Rightarrow R_{T1} = 462\Omega$$

$$T2 = -60 + (60 + 150)/2 = 45^\circ C \Rightarrow R_{T2} = 1161\Omega$$

$$T3 = +150^\circ C \Rightarrow R_{T3} = 2237\Omega$$

Substituting these values gives:

$$R_1 = 2826\Omega$$

(c) What is the sensitivity of the sensor circuit (dR/dT) shown in Figure 1 at a temperature of 25°C?

Answer: The replacement resistance of the two resistors is equal to:

$$R_o = \frac{R_1 R_T}{R_1 + R_T}$$

Use the quotient rule to compute the derivative:

$$\frac{d}{dx}\frac{j(x)}{h(x)} = \frac{j'(x)h(x) - j(x)h'(x)}{(h(x))^2}$$

The sensitivity is thus equal to:

$$\frac{dR_o}{dT} = \frac{R_1(R_1 + R_T) - R_1 R_T}{(R_1 + R_T)^2}\frac{dR_T}{dT}$$

$$\Rightarrow \frac{dR_o}{dT} = \frac{(R_1)^2}{(R_1 + R_T)^2}\frac{dR_T}{dT}$$

From question (a) we know that at T=25°C holds:

$$\left.\frac{dR_T}{dT}\right|_{T=25^\circ C} = \frac{2.3}{298.15K} R_{25} \left(\frac{298.15K}{298.15K}\right)^{1.3} = (0.0077R_{25})/K$$

The sensitivity of the circuit is therefore equal to:

$$\frac{dR_o}{dT} = \frac{(R_1)^2}{(R_1 + R_{25})^2}(0.0077R_{25})/K$$

At T=25°C holds (see question b):

$$R_{25} = 1000\Omega$$

$$R_1 = 2826\Omega$$

The sensitivity of the circuit is thus equal to:

$$\frac{dR_o}{dT} = 4.2\Omega/K$$

In comparison with the sensitivity of the sensor in isolation ($7.7\Omega/K$) this is a reduction of 45%. The non-linearity of the circuit is however much better compared to the sensor in isolation.

Exercise 2: NTC thermistor

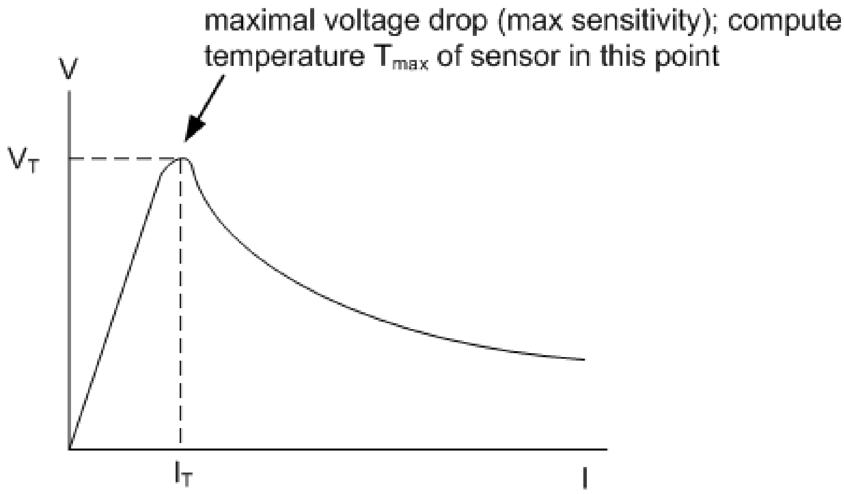
You want to perform a number of measurement with an NTC thermistor $R_T = R_0 e^{B(1/T-1/T_0)}$ for which it holds that $R_T = 10$ kΩ at 25°C. The dissipation constant δ of this sensor is equal to 0.14 mW/K in non-moving air at 25°C. In addition, it holds for this resistance that its ratio between the resistance at 25°C and 125°C is equal to: $R_{25}/R_{125} = 19.8$.

(a) The voltage drop V_T across the thermistor depends on the current I_T through the thermistor and the value of its resistance R_T. When the current through the sensor is limited, there exists an almost linear relation between V_T and I_T. When the current increases, the self-heating effect will result in an increasingly smaller voltage drop across the sensor. At a certain moment, the voltage drop over the sensor will even start to decrease when the current is increased even further. Show that the temperature of the sensor is equal to the following equation when the maximal voltage drop over the sensor occurs:

$$T_{max} = \frac{B - \sqrt{B^2 - 4BT_a}}{2}$$

, with T_a the environmental temperature and T_{max} the temperature of the thermistor.

Answer: The relation between the current through the sensor and the voltage drop across the sensor looks as follows:



(The temperature T_{max} is the temperature which the sensor reaches at the point iin which the voltage drop across V_T the sensor is maximal. Note that this is not the maximal temperature which the sensor can reach. This temperature is much higher. If you increase the current I further, the self-heating effect will continue to increase and therefore also T.)

The power dissipation of this sensor in steady-state is equal to:

$$P_D = I_T^2 R_T = \delta \Delta T = \delta (T - T_a) = \frac{V_T^2}{R_T}$$

It holds for the thermistor:

$$R_T = R_0 e^{B(1/T - 1/T_0)}$$

Substituting this in the equation for the power dissipation gives:

$$V_T^2 = \delta (T - T_a) R_0 e^{B(1/T - 1/T_0)}$$

The temperature at which the voltage drop is maximal (derivative V_T^2 to T equal to 0):

$$\frac{dV_T^2}{dT} = 0$$

$$\Rightarrow \frac{dV_T^2}{dT} = \delta R_0 e^{B(1/T - 1/T_0)} - \frac{B\delta (T - T_a) R_0 e^{B(1/T - 1/T_0)}}{T^2} = 0$$

$$\Rightarrow \delta R_T - \frac{B\delta (T - T_a) R_T}{T^2} = 0$$

Dividing by δR_T gives:

$$\Rightarrow 1 - \frac{B}{T^2} (T - T_a) = 0$$

$$\Rightarrow T^2 - BT + BT_a = 0$$

$$\Rightarrow T = \frac{B \pm \sqrt{B^2 - 4BT_a}}{2}$$

The equation for T has two solutions. The solution corresponding to the temperature with the maximal voltage drop is given by:

$$\Rightarrow T = T_{max} = \frac{B - \sqrt{B^2 - 4BT_a}}{2}$$

(Note that the temperature at which the maximal voltage drop occurs does not depend on the resistance. This temperature depends only on the characteristic temperature of the device and the environmental temperature. So if you keep the voltage drop at its maximum (e.g., using some control circuit) then you can derive the environmental temperature from the voltage drop).

(b) What is the maximal voltage drop V_T across the sensor if the sensor is placed in an environment with non-moving air with a temperature of 35°C?

Answer: As a first step you have to determine the characteristic temperature B of the sensor:

$$B_{T_1/T_2} = \frac{\ln(R_2/R_1)}{(1/T_2 - 1/T_1)}$$

$$\Rightarrow B_{25/125} = \frac{\ln(1/19.8)}{(1/(273K + 125K) - 1/(273K + 25K))} = 3541K$$

The temperature of the thermistor at which the maximal voltage drop occurs is then equal to:

$$T_{max} = \frac{B - \sqrt{B^2 - 4BT_a}}{2} = \frac{(3451K) - \sqrt{(3451K)^2 - 4(3451K)(273 + 35K)}}{2} = 341K$$

The resistance of the thermistor at this temperature is equal to:

$$R_T = R_0 e^{B(1/T - 1/T_0)} = (10k\Omega)e^{(3541K)(1/(341K) - 1/(273K + 25K))} = 2235\Omega$$

The dissipated power in the thermistor is given by:

$$P_D = I_T^2 R_T = \delta\,(T_{max} - T_a)$$

$$\Rightarrow I_T = \sqrt{\frac{\delta\,(T_{max} - T_a)}{R_T}}$$

$$\Leftrightarrow I_T = \sqrt{\frac{(0.14mW/K)\,(341K - (273K + 35K))}{2235\Omega}} = 1.4mA$$

The maximal voltage drop is thus equal to:

$$V_T = I_T R_T = (1.4mA)(2235\Omega) = 3.2V$$

(c) You want to use the thermistor for a certain application around the set-point $T_0 = 25$°C. It is therefore important that the response of the sensor is linearised around this point. To achieve this goal you will use the circuit shown below. Determine the value of the resistors R_1 and R_2 such that the equivalent resistance (transfer function) of the circuit shown an inflection point ("kantelpunt" in Dutch) at T_0 and a sensitivity of -4 Ω/°C.

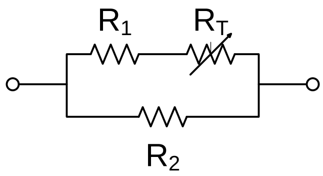


Figure 2: NTC linearisation.

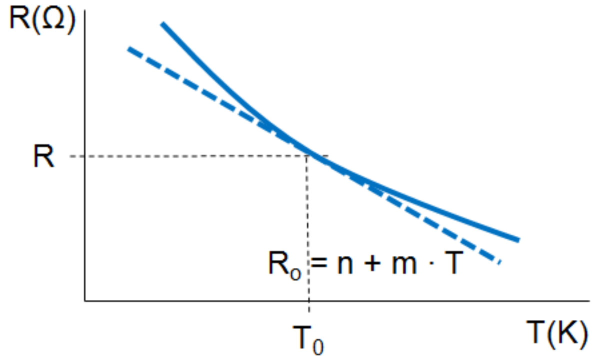
Answer: The resistance of the circuit is equal to:

$$R = (R_1 + R_T)//R_2 = \frac{(R_1 + R_T)R_2}{R_1 + R_T + R_2}$$

The resistors R_1 and R_2 have a fixed ratio with respect to the resistance of the thermistor at its reference temperature. Hence, we can write $R_1 = aR_0$ and $R_2 = bR_0$.

$$R = \frac{(aR_0 + R_T)bR_0}{aR_0 + R_T + bR_0}$$

The question now is to determine values for a and b, such that the transfer function of the circuit matches with the following figure. The inflection point of this transfer function is positioned at T_0 and the transfer function has a sensitivity (slope) at this point of m (= -4 Ω/°C).



Since it is required that the transfer function shows an inflection point at T_0, it must hold that the second derivative of R to T is equal to 0 in this point (T_0). The first derivative is equal to:

$$\frac{dR}{dT} = bR_0 \frac{(aR_0 + R_T + bR_0)\left(-B/T^2\right)R_T - (aR_0 + R_T)\left(-B/T^2\right)R_T}{(aR_0 + R_T + bR_0)^2}$$

$$\Leftrightarrow \frac{dR}{dT} = b^2 R_0^2 \frac{\left(-B/T^2\right)R_T}{(aR_0 + R_T + bR_0)^2}$$

The second derivative is thus equal to:

$$\frac{d^2R}{dT} = b^2 R_0^2 \frac{R_T \frac{B^2}{T^4}\left(\left(\frac{2T}{B} + 1\right)(aR_0 + R_T + bR_0) - 2R_T\right)}{(aR_0 + R_T + bR_0)^3}$$

The requirement is:

$$\left.\frac{d^2R}{dT}\right|_{T=T_0} = 0$$

This requirement is fulfilled when:

$$\left(\frac{2T_0}{B}+1\right)(aR_0+R_T+bR_0)-2R_T=0$$

At $T=T_0$ it holds that $R_T=R_0$. The requirement can thus be written as:

$$\left(\frac{2T_0}{B}+1\right)(aR_0+R_0+bR_0)-2R_0=0$$

Dividing by R_0 results in:

$$\left(\frac{2T_0}{B}+1\right)(a+1+b)-2=0$$

Re-writing this expression gives:

$$a+b=\frac{B-2T_0}{B+2T_0}$$

The sensitivity of the circuit in the inflection point T_0 should be equal to m = -4Ω/°C. To satisfy this requirement it must hold:

$$\left.\frac{dR}{dR}\right|_{T=T_0}=m$$

We computed the first derivative of R to T before. This results in:

$$\frac{dR}{dT}=b^2R_0^2\frac{\left(-B/T^2\right)R_0}{\left(aR_0+R_0+bR_0\right)^2}=m$$

Dividing the numerator and denominator by R_0^2 gives:

$$\Leftrightarrow b^2\frac{\left(-B/T^2\right)R_0}{\left(a+1+b\right)^2}=m$$

It holds:

$$a+b=\frac{B-2T_0}{B+2T_0}$$

Substituting this gives:

$$b^2\frac{\left(-B/T^2\right)R_0}{\left(1+\frac{B-2T_0}{B+2T_0}\right)^2}=m$$

Solving this equation yields:

$$b=\frac{2T_0}{B+2T_0}\sqrt{\frac{-mB}{R_0}}$$

It must therefore hold for a:

$$a=\frac{B-2T_0}{B+2T_0}-b$$

Substituting $B=3541$, $T_0=273K+25K$ and $R_0=10$ kΩ results in:

$$a=0.54, b=0.17$$

It must therefore hold:

$$R_1=5.4k\Omega, R_2=1.7k\Omega$$

(d) Many electrical circuits (e.g. op-amps, integrated circuits) have a positive temperature coefficient. An NTC thermistor is a cheap component to compensate this temperature dependency. To realize this temperature compensation, you can use the circuit shown below. The resistor $R_C = (1k\Omega)(1 + 0.004(T - 273K))$ models the circuit with a positive temperature coefficient. What value should R_1 have such that the total change in the resistance of the circuit around a temperature $T = T_0 = 25°\text{C}$ becomes independent of a small change in the temperature?

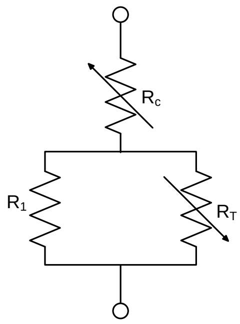


Figure 3: Temperature compensation using an NTC thermistor.

Answer: The equivalent resistance of the circuit is equal to:

$$R = R_1//R_T + R_c = \frac{R_1 R_T}{R_1 + R_T} + R_c$$

It must hold:

$$\left.\frac{dR}{dT}\right|_{T=T_0} = 0$$

(see question 1(c)):

$$\Rightarrow \frac{dR}{dT} = \frac{(R_1)^2}{(R_1 + R_T)^2}\frac{dR_T}{dT} + \frac{dR_c}{dT} = 0$$

$$\Leftrightarrow \frac{(R_1)^2}{(R_1 + R_T)^2}\frac{-B}{T^2}R_T + 4\Omega/K = 0$$

At $T = T_0$ it holds that $R_T = R_0$.

$$\Rightarrow \frac{(R_1)^2}{(R_1 + R_0)^2}\frac{-B}{T_0^2}R_0 + 4\Omega/K = 0$$

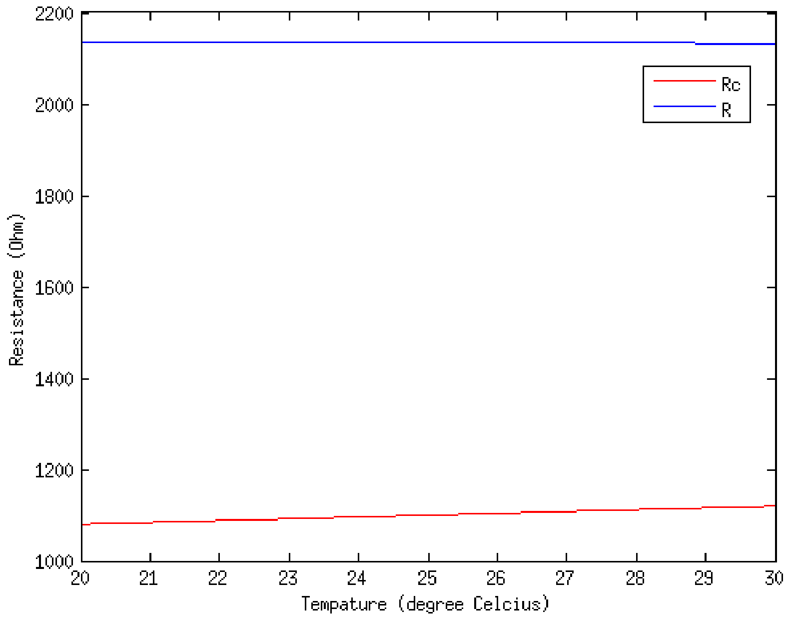
Solving this equation yields:

$$R_1 = \frac{2R_0T_0}{\sqrt{BR_0} - 2T_0}$$

Substituting $B = 3541$, $T_0 = 273K + 25K$ and $R_0 = 10$ kΩ yields:

$$R_1 = 1.2k\Omega$$

The graph shown below shows R_c and R for a small temperature change around T_0.



This figure clearly shows that the temperature dependency of the circuit is much less compared to a solution in which no temperature compensation is used.