Sensing, Computing, Actuating
Lecture 11 - Inductive Sensors and Demodulation

Exercise 1: Measuring inclination

An LVDT sensor is used to measure the angle Θ of a crane. A load of 10 kg is connected to the core of the LVDT sensor. The outer casing of the LVDT sensor is connected to the arm of the crane. The mass is connected to the frame of the sensor using a spring. The mass can move in the longitudinal direction of the axis. This is graphically depicted in the figure below. The primary winding of the LVDT is connected to a supply voltage that generates a sinusoidal voltage with an amplitude of 5V. The LVDT has a sensitivity of 100 mV/(mm/V). The spring constant k is equal to 200 N/cm.

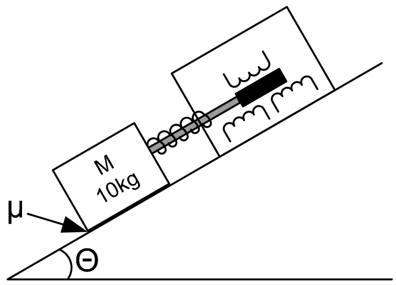


Figure 1: Meausing inclination using an LVDT.

(a) What is the output voltage of the sensor in terms of the angle Θ? (You may ignore the friction between the mass and the arm of the crane.)

Answer:

It holds for the spring: $F = -k \cdot x$ (Hooke's law).
The gravitation exercises a force $F = m \cdot g \cdot sin\left(\Theta\right)$ on the mass.
The displacement of the core is therefore equal to:

$$-k \cdot x + m \cdot g \cdot sin\left(\Theta\right) = 0$$
$$\Leftrightarrow k \cdot x = m \cdot g \cdot sin\left(\Theta\right)$$
$$\Leftrightarrow x = \frac{m \cdot g \cdot sin\left(\Theta\right)}{k}$$

The output voltage of the LVDT sensor is equal to:

$$v_o = S \cdot x \cdot V_i \cdot sin\left(\omega t\right)$$

, with S the sensitivity of the sensor. When we substitute all given values, we find:

$$v_o = 100mV/(mm/V) \cdot \frac{10kg \cdot 9.8m/s^2 \cdot sin\left(\Theta\right)}{200N/cm} \cdot 5V \cdot sin\left(\omega t\right)$$

$$\Leftrightarrow v_o = 100mV/(mm/V) \cdot \frac{10kg \cdot 9.8m/s^2 \cdot sin\left(\Theta\right)}{20000N/m} \cdot 5V \cdot sin\left(\omega t\right)$$

$$\Leftrightarrow v_o = 100mV/(mm/V) \cdot 4.9mm \cdot sin\left(\Theta\right) \cdot 5V \cdot sin\left(\omega t\right)$$

$$\Leftrightarrow v_o = 2.5V \cdot sin\left(\Theta\right) \cdot sin\left(\omega t\right)$$

(b) In the previous question you have ignored the friction between the mass and the arm of the crane. Assume now that the (static) friction is equal to μ. What is the relation between the displacement x of the mass, the angle Θ and the friction μ? (Hint: give x as a function of Θ and μ.)

Answer: The force due to the friction is equal to:

$$F = \mu \cdot F_N = \mu \cdot m \cdot g \cdot cos\left(\Theta\right)$$

There are two situation possible. Gravitation and friction work in the same direction (when the angle Θ becomes smaller). Or gravitation and friction work in opposite directions (when the angle Θ becomes bigger).

Angle Θ smaller:

$$k \cdot x = m \cdot g \cdot sin\left(\Theta\right) + \mu \cdot m \cdot g \cdot cos\left(\Theta\right)$$

$$\Leftrightarrow x = m \cdot g \cdot \frac{sin\left(\Theta\right) + \mu \cdot cos\left(\Theta\right)}{k}$$

Angle Θ bigger:

$$\Leftrightarrow x = m \cdot g \cdot \frac{sin\left(\Theta\right) - \mu \cdot cos\left(\Theta\right)}{k}$$

(c) Does the sensor show hysteresis when we consider the friction? (Explain your answer.)

Answer: Yes. The output voltage of the sensor depends on the situation (increasing or decreasing angle).

(d) A certain application requires that the output voltage of the LVDT sensor is amplified. For this purpose, the sensor from Figure 1 is connected to an instrumentation amplifier. The electrical equivalent circuit is shown in Figure 2.

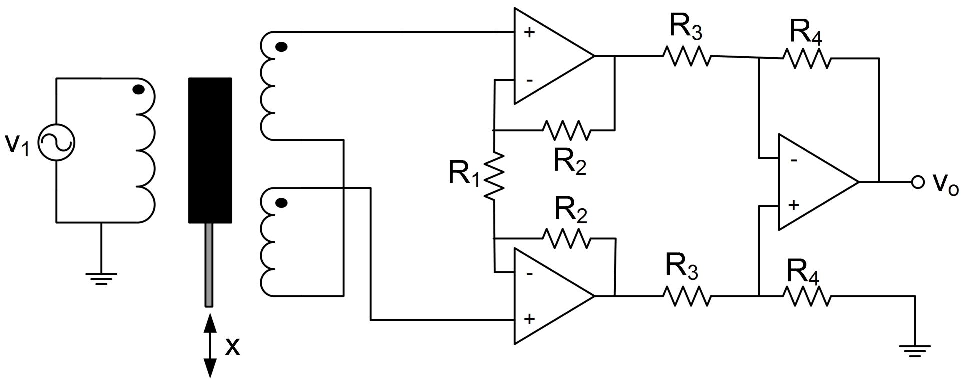


Figure 2: Amplification of the LVDT output signal.

The resistors R_2, R_3 and R_4 have a resistance of 15 kΩ. What value shoudl the resistance R_1 have such that the amplitude of the output voltage v_o is equal to 0 V when $\Theta = 0°$ and 5 V when $\Theta = 90°$? (You may ignore the friction between the mass and the crane.)

Answer:
It holds for the instrumentation amplifier:

$$v_o = \left(1 + \frac{2R_2}{R_1}\right) \frac{R_4}{R_3} \left(v_2 - v_1\right) = \left(1 + G\right) k \left(v_2 - v_1\right)$$

The output voltage of the LVDT sensor is equal to:

$$v_2 - v_1 = 2.5V \cdot sin\,(\Theta) \cdot sin\,(\omega t)$$

It must hold: $V_o = 0V$ when $\Theta = 0°$. This constraint is always met.

It must also hold: $V_o = 5V$ when $\Theta = 90°$. This constraint is met when:

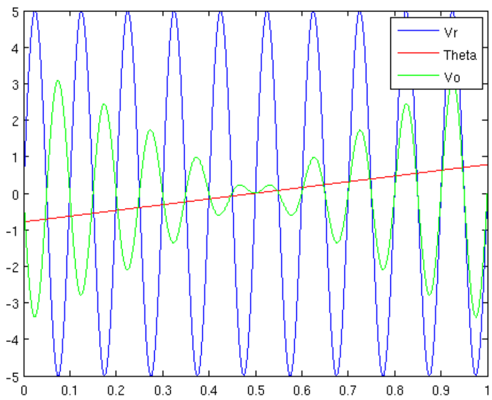
$$5V = (1 + G)\,k \cdot 2.5V$$
$$\Leftrightarrow 2 = 1 + \frac{2R_2}{R_1}$$
$$\Leftrightarrow 1 = \frac{2R_2}{R_1}$$
$$\Leftrightarrow R_1 = 30k\Omega$$

(e) The arm of the crane moves in 1 second from an angle $\Theta = -45°$ to an angle $\Theta = +45°$. Assume that this movement happens with a constant speed. A sinusoidal voltage v_1 with a frequency of 10 Hz and an amplitude of 5V has been placed on the primary winding of the LVDT sensor. Draw the excitation voltage on the primary winding ($v_1(t)$), the angle of the crane ($\Theta(t)$), and the output voltage of the instrumentation amplifier ($v_o(t)$). (Clearly show on each axis the dimension and scale. Ignore the friction between the mass and the crane.)

Answer: A movement from an angle $\Theta = -45°$ to an angle $\Theta = +45°$ is equivalent to a movement from $\Theta = -\pi/4$ to $\Theta = \pi/4$. The output voltage of the instrumentation amplifier is therefore equal to:

$$v_o = 5V \cdot sin\,(\Theta) \cdot sin\,(\omega t)$$

This follows from the result of question (a) and the amplification factor (2x) as computed in the previous question.



The vertical axis is in volt and radials. The horizontal axis is in seconds.

(f) The output of the instrumentation amplifier from Figure 2 is connected to a single-sided rectifier with low-pass filter as shown in Figure 3. Draw the output voltage ($v_o(t)$) of the circuit when the arm is kept still at an angle of 45°.

Answer: The diode conducts when the voltage across it is positive. The low-pass filter takes the average value from the positive voltage across the diode (approximately $1/\pi \cdot V_{peak}$).

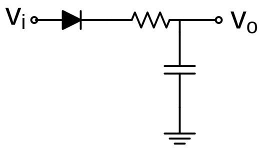
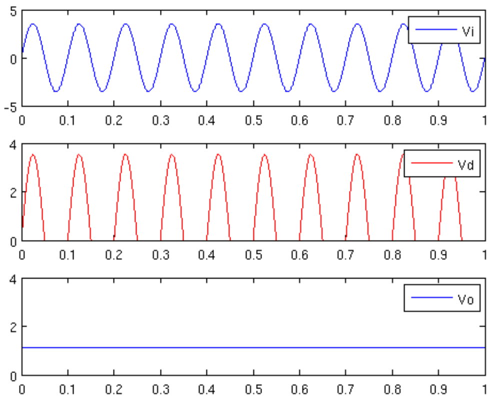


Figure 3: Single-sides rectifier with low-pass filter



(g) Can you reconstruct the direction (positive or negative angle) from the output signal ($v_o(t)$) of the circuit with the single-sided rectifier? (Explain your answer.)

Answer: No. The rectifier has the same behaviour when the core moves in the positive or negative direction.

(h) Instead of the instrumentation amplifier from Figure 2 and the single-sided rectifier, you may connect the LVDT sensor from Figure 1 directly to a double-sided rectifier. In addition, the middle connector of the secondary windings is connected to ground. The electrical schematic of the sensor and its signal conditioning circuit is shown in Figure 4. Draw the output voltage ($v_o(t) = v_2(t) - v_1(t)$) of the circuit when the arm is kept still at an angle of 45°.

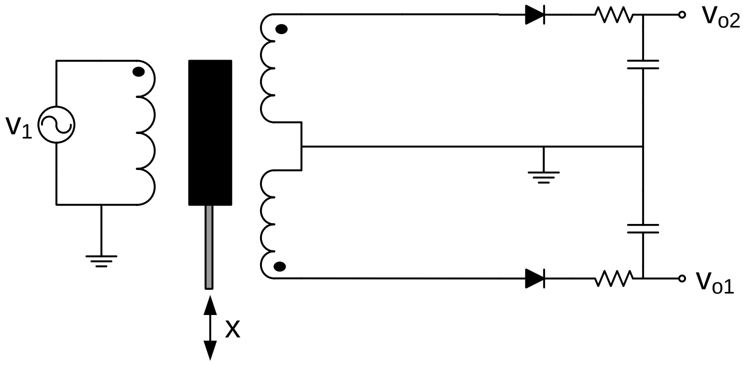
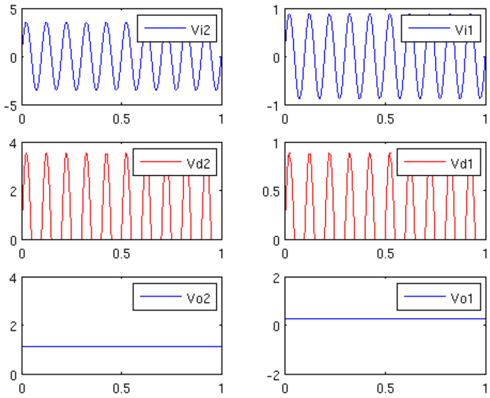


Figure 4: Double-sided rectifier with low-pass filter.

Answer: The coupling between the primary coil and the top secondary coil will be bigger then the coupling between the primary coil and the bottom secondary coil. As a result, the rectified voltage on v_{o2} will be larger then the rectified voltage on v_{o1}. The output voltage v_o is the difference between these two voltages.



(i) Can you reconstruct the direction (positive or negative angle) from the output signal ($v_o(t)$) of the circuit with the double-sided rectifier? (Explain your answer.)

Answer: Yes. In the previous question we have assumed that with an angle of $+45°$ the coupling between the primary coil and the top secondary coil will be bigger then the coupling between the primary coil and the bottom secondary coil. This resulted in a positive output voltage. When the angle is $-45°$, the situation will be exactly opposite and the output voltage will become negative. Looking at the sign of the output voltage it is possible to recover the direction.